(1) Find the rank and a basis for the row space and for the column space of the given matrices.

(i) \[
\begin{bmatrix}
1 & -2 \\
0 & 0 \\
-3 & 6 \\
\end{bmatrix}
\]  
(ii) \[
\begin{bmatrix}
0 & -2 & 1 & 3 \\
1 & 4 & 0 & 7 \\
5 & 5 & 5 & 5 \\
\end{bmatrix}
\]  
(iii) \[
\begin{bmatrix}
8 & 0 & 4 \\
0 & 2 & 0 \\
4 & 0 & 2 \\
0 & 4 & 0 \\
\end{bmatrix}
\]

(2) Determine whether the vector \( \mathbf{w} = (1, 5, -2) \) is in the span of the vectors \( \mathbf{v}_1 = (1, -1, 1), \mathbf{v}_2 = (1, 1, 0), \mathbf{v}_3 = (1, -3, 2), \mathbf{v}_4 = (1, 3, -1) \).

(3) Determine whether the vector \( \mathbf{w} = (3, 5, 1) \) is in the span of the vectors \( \mathbf{v}_1 = (1, 1, 1), \mathbf{v}_2 = (2, 3, 1), \mathbf{v}_3 = (0, -1, 1) \).

(4) Verify that the following system is inconsistent by finding the rank of the matrices \( \mathbf{A} \) and \( \mathbf{A} \mathbf{b} \).
\[
\begin{align*}
x + y + z &= 1 \\
2x + 2y + 2z &= 4
\end{align*}
\]

(5) Express the general solution of the following homogeneous system as a linear combination of linearly independent vector. Find the dimension of the solution space.
\[
\begin{align*}
x + y - 2z &= 0 \\
-4w - x - y + 2z &= -4 \\
-2w + 3x + 3y - 6z &= -2
\end{align*}
\]