1. Determine whether $x = 0$ is an ordinary point/regular or irregular singular point for the following differential equations:

(i) The Bessel Equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$ where $\alpha$ is a constant.

(ii) $xy'' + (1 - \cos x)y' + x^2y = 0$.  \[4\]

2. Consider the matrix $A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of $A$. \[4\]

(b) Find the fundamental matrix $\Phi(t)$ of the differential system $x' = Ax$. \[3\]

(c) Find the inverse $\Phi^{-1}(t)$ of the fundamental matrix $\Phi(t)$. \[3\]

(d) Find a particular solution of the nonhomogeneous system $x' = Ax + g(t)$ where $g(t) = \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix}$. \[4\]

3. Find the general solution of the first order homogeneous system $x' = Ax$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}$. \[6\]

4. If $\begin{pmatrix} 2 + 2i \\ -1 \end{pmatrix}$ is an eigenvector for a matrix $A$, corresponding to an eigenvalue $\lambda = 2i$, give two linearly independent real valued solutions of the system of ODE $x' = Ax$. \[5\]
5. Find the solution of the Cauchy-Euler equation \( t^2 y'' - t y' + 2y = 0 \), satisfying the initial values \( y(1) = 1, \ y'(1) = 0 \). \[6\]

6. If \( y_1, y_2 \) are two linearly independent solutions of \( y'' + ay' + by = 0 \) prove that \( y_3 = y_1 + y_2 \) and \( y_4 = y_1 - y_2 \) are also linearly independent. \[5\]

7. Find the solution of the logistic equation \( y' = y - y^2 \), \( r > 0, k > 0 \), are given constants and \( y(0) = 2 \). \[5\]

8. Use Laplace transform method to find the solution of the IVP \( y'' + y = \sin t \), \( y(0) = 1, y'(0) = -1 \). \[5\]

Education is not mere collection of facts, but the concentration of mind. The end (result) of Education is Character.