Practice Problems

MTH 2201 6/7/2007

1. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system. Assume that the variables are named \(x_1, x_2, \ldots\), from left to right.

\[
\begin{pmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 7
\end{pmatrix} \quad \begin{pmatrix}
1 & 0 & 0 & -7 & 8 \\
0 & 1 & 0 & 3 & 2 \\
0 & 0 & 1 & 1 & -5
\end{pmatrix} \quad \begin{pmatrix}
1 & 2 & 0 & 2 & -1 & 3
\end{pmatrix}
\]

2. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system by reducing the matrix to reduced row echelon form. Assume that the variables are named \(x_1, x_2, \ldots\), from left to right.

\[
\begin{pmatrix}
1 & -3 & 4 & 7 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & 5
\end{pmatrix} \quad \begin{pmatrix}
1 & 7 & -2 & 0 & -8 & -3 \\
0 & 0 & 1 & 1 & 6 & 5 \\
0 & 0 & 0 & 1 & 3 & 9 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

3. Solve the linear system by Gauss-Jordan elimination:

\[
\begin{align*}
x_1 + x_2 + 2x_3 &= 8 \\
-2x_1 - x_2 + 3x_3 &= 1 \\
3x_1 - 7x_2 + 4x_3 &= 10
\end{align*}
\]

\[
\begin{align*}
3x_1 + 2x_2 - x_3 &= -15 \\
5x_1 + 3x_2 + 2x_3 &= 0 \\
3x_1 + x_2 + 3x_3 &= 11 \\
6x_1 - 4x_2 + 2x_3 &= 30
\end{align*}
\]

4. Solve the following linear homogeneous system:

\[
\begin{align*}
2x_1 + x_2 + 3x_3 &= 0 \\
x_1 + 2x_2 + x_3 &= 0 \\
x_2 + 2x_3 &= 0
\end{align*}
\]

\[
\begin{align*}
3x_1 + x_2 + x_3 + x_4 &= 0 \\
5x_1 - x_2 + x_3 - x_4 &= 0
\end{align*}
\]

5. Determine the values of \(a\) for which the system has no solutions, exactly one solution, or infinitely many solutions:

\[
\begin{align*}
x &+ 2y &+ 3z &= 4 \\
3x &- y &+ 5z &= 2 \\
4x &+ y &- 14z &= a &+ 2
\end{align*}
\]

6. What relationship must exist between \(a, b,\) and \(c\) for the following linear system to be consistent?

\[
\begin{align*}
x &+ y &+ 2z &= a \\
x &+ z &= b \\
2x &+ y &+ 3z &= c
\end{align*}
\]
7. Let \( A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \) and \( B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \).

(i) Find the second column vector of \( AB \).
(ii) Find the third row vector of \( AB \).
(iii) Find \( tr(AB) - tr(A)tr(B) \).

8. (i) Show that if \( A \) has a row of zeros and \( B \) is any matrix for which \( AB \) is defined, then \( AB \) also has a row of zeros.
(ii) Show that if \( B \) and \( C \) have two equal columns, and \( A \) is any matrix for which \( AB \) and \( AC \) are defined, then \( AB \) and \( AC \) also have two equal columns.

9. How many \((3 \times 3)\) matrices \( A \) can you find such that \( A \) has constant entries and

\[
A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \\ 0 \end{bmatrix},
\]

for all real values of \( x, y \) and \( z \)?