Homework 4 : Second Order Linear ODE

1. Find all solutions of $y'' + 2iy' + y = 0$

2. Find all solutions of $y'' + (3i - 1)y' - 3iy = 0$

3. Find the general solution of

   (i) $4y'' + y' = 0$.  
   (ii) $y'' + 8y' + 16y = 0$.  
   (iii) $8y'' + 2y' - y = 0$.  
   (iv) $y'' - y = 0$.  
   (v) $y'' - 4y'' - 5y' = 0$.  
   (vi) $d^2y/dx^2 - 16dy/dx = 0$.  
   (vii) $16d^4y/dx^4 + 24d^2y/dx^2 + 9y = 0$.

4. Consider $y'' + y' - 6y = 0$.
   (i) Compute the solution $\phi$ satisfying $\phi(0) = 1, \phi'(0) = 0$.
   (ii) Compute the solution $\psi$ satisfying $\psi(0) = 0, \psi'(0) = 1$.

5. Find all solutions $\phi$ of $y'' + y = 0$ satisfying $\phi(0) = 1, \phi(\pi/2) = 2$.

6. Let $\phi$ be a solution of the equation $y'' + a_1y' + a_2y = 0$, where $a_1, a_2$ are constants. If $\psi(t) = e^{(a_1/2)t}\phi(t)$. Show that $\psi$ satisfies the DE $y'' + ky = 0$, where $k$ is some constant.

7. Find the solutions to the following IVPs:
   (i) $y'' + (3i - 1)y' - 3iy = 0, \ y(0) = 2, y'(0) = 0$.
   (ii) $y'' + 10y = 0, \ y(0) = \pi, y'(0) = \pi^2$.

8. Suppose $\phi$ is a function having a continuous derivative on $0 \leq x < \infty$ such that $\phi'(x) + 2\phi(x) \leq 1$ for all such $x$, and $\phi(0) = 0$. Show that $\phi(x) < 1/2$ for all $x \geq 0$.

9. Determine the values of $\alpha$, for which all solutions of $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$, tend to zero as $t \to 0$.  

1
10. An equation of the form $t^2y'' + \alpha ty' + \beta y = 0, t > 0,$ is called an Euler Equation. Show that $x = \ln t,$ transforms the equation to an ODE with constant coefficients. Use this result to solve, for $t > 0, t^2y'' + 3ty' + 54y = 0.$

You may also solve the following problems from your text.

Exercise 3.4 :
Problems 1, 7, 13, 17, 21, 35, 37, 41, 43, 53, 55.

Exercise 3.5 :
5, 15, 21, 23, 25.