Homework 2: Linear Equations

1. Find the general solution of the given ODE and give the largest interval on which the solution is defined.
   
   (i) \( y' + 3t^2y = t^2 \).  
   (ii) \( t \frac{dy}{dt} - y = t^2 \sin t \).
   
   (ii) \( t^2y' + t(t + 2)y = e^t \).  
   (iv) \( (\cos t) \frac{dy}{dt} + (\sin t)y = 1 \).
   
   (v) \( \frac{dr}{d\theta} + r \sec \theta = \cos \theta \).

2. Find the solutions to the following IVPs:
   
   (i) \( \frac{dT}{dt} = K(T - T_m), \ T(0) = T_0, \ K, T_m, T_0 \) are constants.
   
   (ii) \( (t + 1) \frac{dx}{dt} + x = \ln t, \ x(1) = 10 \).
   
   (iii) \( \frac{dy}{dt} + y = f(t), \ f(t) = \begin{cases} 
   1, & 0 \leq t \leq 1 \\
   -1, & t > 1 
   \end{cases} , \ y(0) = 1 \).

3. Solve \( \frac{dy}{dt} - 2ty = 2, \ y(0) = 1 \), in terms of \( \text{erf}(t) \).

4. Solve \( \frac{dy}{dt} - 2ty = -1, \ y(0) = \frac{\sqrt{\pi}}{2} \), in terms of \( \text{erfc}(t) \).

5. Define \( Si(t) = \int_0^t \frac{\sin u}{u} du \).

   Show that the solution of the IVP \( t^3y' + 2t^2y = 10 \sin t, \ y(1) = 0 \) is \( y(t) = 10t^{-2}[Si(t) - Si(1)] \).