1. Let $T_A$ be the transformation whose standard matrix is $A$. Find a vector $x$ if any, whose image under $T_A$ is $b$.

   (i) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 2 & 5 & -3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

   Solution: $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -6 \\ 3 \\ 1 \end{bmatrix}, t \in \mathbb{R}$.

2. If $T$ is a linear transformation such that $T(1,0,0) = (1,1,2), T(0,1,0) = (1,1,2)$ and $T(0,0,1) = (0,3,1)$. Find $[T]$.

3. Find the standard matrix for $T(x_1,x_2) = (-x_1+x_2,x_2)$. Check your answer by calculating $T((-1,4))$ directly.

4. Use matrix multiplication to find the image of the vector $x = (-2,1)$ under the transformation (i) reflection about the line $y = -x$.

5. Use matrix multiplication to find the image of the vector $x = (3,4)$ under rotation of (i) 45 degrees about the origin (ii) -60 degrees about the origin.

6. Use matrix multiplication to find the image of the point $(4,3)$ under the transformation of reflection about the line through origin that makes an angle of $\theta = \frac{\pi}{3}$ with the positive $x$ axis.

7. Confirm that the matrix $A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{6}} \end{bmatrix}$ is orthogonal and find its inverse.

8. Confirm that the matrix $A = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ is orthogonal and determine whether multiplication by $A$ is rotation or a reflection about a line through origin. As appropriate, find the rotation angle or the angle that line makes with the positive $x$ axis.