1. Show that the vectors $\mathbf{u} = (1, 4, 4)$, $\mathbf{v} = (-1, 3, 6)$, $\mathbf{w} = (-3, 2, 8)$ are linearly dependent by expressing one or more vectors as a linear combination of the other two.

2. Find a general solution of the linear system and list a set of vectors that span the solution space:
\begin{align*}
(\text{i}) \quad x_1 + 2x_2 + x_3 + x_4 + x_5 &= 0 \\
2x_1 + 4x_2 - x_3 + x_5 &= 0.
\end{align*}

3. Determine whether the given set of vectors are linearly independent.
   \begin{enumerate}
   \item $\mathbf{v}_1 = (2, 5, 4), \mathbf{v}_2 = (0, -6, 2), \mathbf{v}_3 = (2, 1, 8), \mathbf{v}_4 = (4, -3, 7)$.
   \item $\mathbf{v}_1 = (4, 7, 6, -2), \mathbf{v}_2 = (9, 7, 2, -1), \mathbf{v}_3 = (4, -3, 1, 5), \mathbf{v}_4 = (6, 4, 9, 1), \mathbf{v}_5 = (4, -7, 0, 6)$.
   \end{enumerate}

4. Show that the set $W$ of all vectors of the form $\mathbf{x} = (a, b, 2a, 3b, -a)$ in which $a$ and $b$ are any real numbers is a subspace of $\mathbb{R}^5$ by finding a set of spanning vectors for $W$.

5. For which real values of $\lambda$ do the following vectors form a linearly dependent set in $\mathbb{R}^3$.
   $\mathbf{v}_1 = (\lambda, \frac{1}{2}, \frac{1}{2}), \mathbf{v}_2 = (\frac{1}{2}, \lambda, \frac{1}{2}), \mathbf{v}_3 = (\frac{1}{2}, \frac{1}{2}, \lambda)$.

6. Show that if $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are any vectors in $\mathbb{R}^n$ then the vectors $\mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3, \mathbf{v}_3 - \mathbf{v}_1$ form linearly dependent set.

7. Find a general solution of the system, state the dimension of the solution space and confirm that the row vectors of $A$ are orthogonal to the solution vectors.
\begin{align*}
(\text{i}) \quad x_1 + 5x_2 + x_3 + 2x_4 - x_5 &= 0 \\
-2x_2 - x_3 + 3x_4 + 2x_5 &= 0.
\end{align*}