Practice Problems

MTH 3102 2/5/2007

1. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system. Assume that the variables are named $x_1, x_2, \ldots$, from left to right.

(i) \[
\begin{bmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 7
\end{bmatrix}
\]

(ii) \[
\begin{bmatrix}
1 & 0 & 0 & -7 & 8 \\
0 & 1 & 0 & 3 & 2 \\
0 & 0 & 1 & 1 & -5
\end{bmatrix}
\]

(iii) \[
\begin{bmatrix}
1 & 2 & 0 & 2 & -1 & 3
\end{bmatrix}
\]

2. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system by reducing the matrix to reduced row echelon form. Assume that the variables are named $x_1, x_2, \ldots$, from left to right.

(i) \[
\begin{bmatrix}
1 & -3 & 4 & 7 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & 5
\end{bmatrix}
\]

(ii) \[
\begin{bmatrix}
1 & 7 & -2 & 0 & -8 & -3 \\
0 & 0 & 1 & 1 & 6 & 5 \\
0 & 0 & 0 & 1 & 3 & 9 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

3. Solve the linear system by Gauss-Jordan elimination:

(i) \[x_1 + x_2 + 2x_3 = 8 \quad 3x_1 + 2x_2 - x_3 = -15\]

(ii) \[-x_1 - 2x_2 + 3x_3 = 1 \quad 5x_1 + 3x_2 + 2x_3 = 0\]

(iii) \[3x_1 - 7x_2 + 4x_3 = 10 \quad 3x_1 + x_2 + 3x_3 = 11\]

6x_1 - 4x_2 + 2x_3 = 30

4. Solve the following linear homogeneous system:

(i) \[2x_1 + x_2 + 3x_3 = 0 \quad x_1 + 2x_2 = 0 \quad x_2 + 2x_3 = 0\]

(ii) \[3x_1 + x_2 + x_3 + x_4 = 0 \quad 3x_1 + x_2 + x_3 = 0 \quad 5x_1 - 2x_2 + x_3 - x_4 = 0\]

5. Determine the values of $a$ for which the system has no solutions, exactly one solution, or infinitely many solutions:

(i) \[x + 2y + 3z = 4 \quad 3x - y + 5z = 2 \quad 4x + y - 14z = a + 2\]

6. What relationship must exist between $a, b, c$ for the following linear system to be consistent?

\[x + y + 2z = a \quad x + z = b \quad 2x + y + 3z = c\]
7. Find the values of the constants that make the given equation an identity:

\[
\frac{3x^3 + 4x^2 - 6x}{(x^2 + 2x + 2)(x^2 - 1)} = \frac{Ax + B}{x^2 + 2x + 2} + \frac{C}{x - 1} + \frac{D}{x + 1}
\]

(Hint: Obtain a common denominator on the right and then equate the corresponding coefficients.)

8. Find a quadratic polynomial whose graph passes through the points \((0, 0), (-1, 1)\) and \((1, 1)\).

9. Let \(A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}\) and \(B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}\).
   (i) Find the second column vector of \(AB\).
   (ii) Find the third row vector of \(AB\).
   (iii) Find \(tr(AB) - tr(A)tr(B)\).

10. (i) Show that if \(A\) has a row of zeros and \(B\) is any matrix for which \(AB\) is defined, then \(AB\) also has a row of zeros.
    (ii) Show that if \(B\) and \(C\) have two equal columns, and \(A\) is any matrix for which \(AB\) and \(AC\) are defined, then \(AB\) and \(AC\) also have two equal columns.

11. How many \((3 \times 3)\) matrices \(A\) can you find such that \(A\) has constant entries and

   \[
   A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \\ 0 \end{bmatrix},
   \]

   for all real values of \(x, y\) and \(z\)?

12. A matrix \(S\) is said to be a **square root** of a matrix \(M\) if \(SS = M\).
    Let \(A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\) and \(B = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}\).  
    (i) Find two square roots of \(A\).
    (ii) How many different square roots of \(B\) can you find?