(1) Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system. Assume that the variables are named \( x_1, x_2, \ldots \), from left to right.

\[
\begin{bmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 7
\end{bmatrix} \quad \begin{bmatrix}
1 & 0 & 0 & -7 & 8 \\
0 & 1 & 0 & 3 & 2 \\
0 & 0 & 1 & 1 & -5
\end{bmatrix}
\]

(Solutions: \((-3,0,7), (8 + 7t, 2 - 3t, -5 - t, t)\))

(2) Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system by reducing the matrix to reduced row echelon form. Assume that the variables are named \( x_1, x_2, \ldots \), from left to right.

\[
\begin{bmatrix}
1 & -3 & 4 & 7 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & 5
\end{bmatrix} \quad \begin{bmatrix}
1 & 7 & -2 & 0 & -8 & -3 \\
0 & 0 & 1 & 1 & 6 & 5 \\
0 & 0 & 0 & 1 & 3 & 9 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(3) Solve the linear systems by Gauss elimination:

(i) \[
\begin{align*}
x_1 + x_2 + 2x_3 &= 8 \\
-2x_1 + 3x_2 &= 1 \\
3x_1 - 7x_2 + 4x_3 &= 10
\end{align*}
\]

(ii) \[
\begin{align*}
3x_1 + 2x_2 - x_3 &= -15 \\
5x_1 + 3x_2 + 2x_3 &= 0 \\
3x_1 + x_2 + 3x_3 &= 11 \\
6x_1 - 4x_2 + 2x_3 &= 30
\end{align*}
\]

(4) Solve the following linear homogeneous system:

(i) \[
\begin{align*}
2x_1 + x_2 + 3x_3 &= 0 \\
x_1 + 2x_2 &= 0 \\
x_2 + 2x_3 &= 0
\end{align*}
\]

(ii) \[
\begin{align*}
3x_1 + x_2 + x_3 + x_4 &= 0 \\
5x_1 - x_2 + x_3 - x_4 &= 0
\end{align*}
\]

(Solutions: (i) only trivial solution (ii) \((-\frac{1}{7}t, \frac{1}{7}t - s, s, t)\))
(5) Determine the values of \( a \) for which the system has no solutions, exactly one solution, or infinitely many solutions:

\[
\begin{align*}
x + 2y + 3z &= 4 \\
3x - y + 5z &= 2 \\
4x + y - 14z &= a + 2
\end{align*}
\]

(solution: exactly one solution for every \( a \).)

(6) What relationship must exist between \( a, b, \) and \( c \) for the following linear system to be consistent?

\[
\begin{align*}
x + y + 2z &= a \\
x + z &= b \\
2x + y + 3z &= c
\end{align*}
\]

(solution: \( c = (a + b) \))

(7) Find a cubic polynomial whose graph passes through the points \((-1, -1), (0, 1), (1, 3)\) and \((4, -1)\). (solution: \( p(x) = 1 + \frac{13}{6}x - \frac{1}{6}x^3 \))

(8) Gauss method works by combining the equations in a system to make new equations. Can the equation \(6x - 9y + 5z = -2\) be derived, by a sequence of Gaussian reduction steps, from the following system?

\[
\begin{align*}
2x + y - z &= 4 \\
6x - 3y + z &= 5
\end{align*}
\]

(solution: yes; Find the steps!)

(9) Find all values of \( a, b, c \) for which \( A \) is symmetric.

\[
\begin{bmatrix}
2 & a - 2b + 2c & 2a + b + c \\
3 & 5 & a + c \\
0 & -2 & 7
\end{bmatrix}
\]

Solution: \( a = 11, b = -9, c = -13 \).

(10) Four positive integers are given. Select any three of the integers, find their arithmetic average and add this result to the fourth integer. The numbers obtained are 29, 23, 21 and 17. What are the integers given. (Solution: 3, 9, 12 and 21)