Homework 10: Second Order Linear ODE

1. In the problems below, $y_1(x)$ is a solution of the given DE. Use reduction of order method to find the second solution of the DE.

   (a) $y'' - 4y' + 4y = 0; \quad y_1 = e^{2x}.$ (Solution: $y_2 = xe^{2x}$)
   (b) $y'' + 16y = 0; \quad y_1 = \cos 4x.$ (Solution: $y_2 = \cos 4x$)
   (c) $9y'' - 12y' + 4y = 0; \quad y_1 = e^{\frac{2x}{3}}.$ (Solution: $y_2 = xe^{\frac{2x}{3}}$)
   (d) $x^2 y'' - 7xy' + 16y = 0; \quad y_1 = x^4.$ (Solution: $y_2 = x^4 \ln |x|$.)
   (e) $(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0; \quad y_1 = x + 1.$ (Solution: $y_2 = x^2 + x + 2$.)

2. Find the general solution of
   (i) $4y'' + y' = 0.$
   (ii) $y'' + 8y' + 16y = 0.$
   (iii) $8y'' + 2y' - y = 0.$
   (iv) $y'' - y = 0$

3. Consider $y'' + y' - 6y = 0.$
   (i) Compute the solution $\phi$ satisfying $\phi(0) = 1, \phi'(0) = 0.$
   (ii) Compute the solution $\psi$ satisfying $\psi(0) = 0, \psi'(0) = 1.$

4. Find all solutions $\phi$ of $y'' + y = 0$ satisfying $\phi(0) = 1, \phi(\pi/2) = 2$

5. Let $\phi$ be a solution of the equation $y'' + a_1 y' + a_2 y = 0,$ where $a_1, a_2$ are constants. If $\psi(t) = e^{(a_1/2)t} \phi(t).$ Show that $\psi$ satisfies the DE $y'' + ky = 0,$ where $k$ is some constant.

6. Determine the values of $\alpha,$ for which all solutions of $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0,$ tend to zero as $t \to 0.$

7. Find the general solution of
   (i) $y'' - 4y' - 5y = 0$
   (ii) $\frac{d^3 y}{dx^3} - 16 \frac{dy}{dx} = 0$
   (iii) $16 \frac{d^4 y}{dx^4} + 24 \frac{d^2 y}{dx^2} + 9y = 0.$