CHAPTER 4

FLIGHT PERFORMANCE

This chapter is an introduction to the flight performance of rocket-propelled vehicles such as spacecraft, space launch vehicles, missiles, or projectiles. It presents the subjects from a rocket propulsion point of view. Rocket propulsion systems provide forces to a flight vehicle and cause it to accelerate (or sometimes to decelerate), overcome drag forces, or change flight direction. Some propulsion systems provide also torques to the flight vehicles for rotation or maneuvers. The flight missions can be classified into several flight regimes: (1) flight within the earth’s atmosphere (air-to-surface missiles, surface-to-surface short-range missiles, surface-to-air missiles, air-to-air missiles, assisted takeoff units, sounding rockets, or aircraft rocket propulsion systems), Refs. 4–1 and 4–2; (2) near space environment (earth satellites, orbital space stations, or long-range ballistic missiles), Refs. 4–3 to 4–9; (3) lunar and planetary flights (with or without landing or earth return), Refs. 4–5 to 4–12; and (4) deep space exploration and sun escape. Each is discussed in this chapter. The chapter begins with the analysis of simple one-dimensional space flight and then treats more complex flight path scenarios and various flying rocket-propelled vehicles. The appendices give conversion factors, atmosphere properties, and a summary of key equations.

4.1. GRAVITY-FREE DRAG-FREE SPACE FLIGHT

This first simple rocket flight analysis applies to an outer space environment, far away from any star, where there is no air (thus no drag) and essentially no significant gravitational attraction. The flight direction is the same as the thrust direction (along the axis of the nozzle), namely, a one-dimensional, straight-line acceleration path; the propellant mass flow \( \dot{m} \) and thus the propulsive thrust \( F \)
remain constant for the propellant burning duration \( t_p \). The thrust force \( F \) has been defined in Eq. 2–17. For a constant propellant flow the flow rate \( \dot{m} \) is \( m_p / t_p \), where \( m_p \) is the total usable propellant mass. From Newton’s second law and for an instantaneous vehicle mass \( m \) and a vehicle flight velocity \( u \)

\[
F = m \frac{du}{dt}
\]

For a rocket where the propellant flow rate is constant and the start or shut-off periods are very brief, the instantaneous mass of the vehicle \( m \) can be expressed as a function of the initial mass of the full vehicle \( m_0 \), the propellant mass \( m_p \), the time at power cutoff \( t_p \), and the instantaneous time \( t \):

\[
m = m_0 - \frac{m_p}{t_p} t = m_0 \left( 1 - \frac{m_p t}{m_0 t_p} \right)
\]

\[
= m_0 \left( 1 - \frac{\zeta t}{t_p} \right) = m_0 \left[ 1 - \left( 1 - \frac{MR}{t_p} \right) \frac{t}{t_p} \right]
\]

Equation 4–3 expresses the vehicle mass in a form useful for trajectory calculations. The vehicle mass ratio \( MR \) (final mass/initial mass or \( m_f / m_0 \)) and the propellant mass fraction \( \zeta \) have been defined by Eqs. 2–7 and 2–8. They are related by

\[
\zeta = 1 - \frac{MR}{1 - \frac{m_f}{m_0}} = 1 - \frac{m_f}{m_0}
\]

A definition of the various masses is shown in Fig. 4–1. The initial mass at takeoff \( m_0 \) equals the sum of the useful propellant mass \( m_p \) plus the empty or final vehicle mass \( m_f \); \( m_f \) in turn equals the sum of the inert masses of the engine system (such as nozzles, tanks, cases, or unused, residual propellant) plus the guidance, control, electronics, and related equipment and the payload. The residual propellant remaining in the propulsion system after thrust termination is not available for combustion and is usually considered to be a part of the final engine mass after operation. This is the liquid propellant trapped in pipe pockets, valve cavities, and pumps or wetting the tank and pipe walls. For solid propellant rocket motors it is the remaining unburned solid propellant, also called slivers, and sometimes also unburned insulation.

For constant propellant mass flow \( m \) and a finite propellant burning time \( t_p \), the total propellant mass \( m_p \) is \( m t_p \), and the instantaneous vehicle mass \( m = m_0 - \dot{m} t \). Equation 4–1 can be written as

\[
du = \frac{F}{m} dt = \frac{(c m)}{m} dt
\]

\[
= \frac{c (m_p / t_p) dt}{m_0 - m_p t / t_p} = \frac{c \zeta / t_p}{1 - \zeta t / t_p} dt
\]

The start period and the shutdown period are very short and can be neglected. Integration leads to the maximum vehicle velocity at propellant burnout \( u_p \) that can be attained in a gravity-free vacuum. When the initial flight velocity \( u_0 \) is not zero \( (u_0 \neq 0) \) it is often called the velocity increment \( \Delta u \):

\[
\Delta u = - c \ln(1 - \zeta) + u_0 = c \ln(m_0 / m_f) + u_0
\]

If the initial velocity \( u_0 \) is assumed to be zero, then the velocity at thrust termination

\[
u_p = \Delta u = - c \ln(1 - \zeta) = - c \ln(m_0 / (m_0 - m_p))
\]

\[
= - c \ln(MR) = c \ln(1 / MR)
\]

\[
= c \ln(m_0 / m_f)
\]

The symbol \( \ln \) stands for the natural logarithm. This \( u_p \) is the maximum velocity increment \( \Delta u \) that can be obtained in a gravity-free vacuum with constant propellant flow, starting from rest with \( u_0 = 0 \). The effect of variations in \( c \), \( I_s \), and \( \zeta \) on the flight velocity increment is shown in Fig. 4–2. An alternate way to write Eq. 4–6 uses \( e \), the base of the natural logarithm:

\[
e^{\Delta u / c} = 1 / MR = m_0 / m_f
\]

The concept of the maximum attainable flight velocity increment \( \Delta u \) in a gravity-free vacuum is useful in understanding the influence of the basic parameters. It is used in comparing one propulsion system or vehicle with another,
radios, guidance system, aerodynamic lifting surfaces, and so on; the remaining 80% is useful propellant. It requires careful design to exceed 0.85; mass fraction ratios approaching 0.95 appear to be the probable practical limit for single-stage vehicles and currently known materials. When the mass fraction is 0.90, then $1/\mathbf{MR} = 0.1$ and $\mathbf{MR} = 10.0$. This marked influence of mass fraction or mass ratio on the velocity at power cutoff, and therefore also the range, not only is true of interplanetary spaceships in a vacuum but applies to almost all types of rocket-powered vehicles. For this reason, importance is placed on saving inert mass on every vehicle component, including the propulsion system.

Equation 4–6 can be modified and solved for the effective propellant mass $m_p$ required to achieve a desired velocity increment for a given initial takeoff mass or a final burnout mass of the vehicle. The final mass consists of the payload, the structural mass of the vehicle, the empty propulsion system mass (which includes residual propellant), plus a small additional mass for guidance, communications, and control devices. Here $m_p = m_0 - m_f$:

$$m_p = m_f (e^{\Delta u/c} - 1) = m_0 (1 - e^{(-\Delta u/c)})$$ \hspace{1cm} (4–8)

The flight velocity increment $\Delta u$ is proportional to the effective exhaust velocity $c$ and, therefore, to the specific impulse. Thus any improvement in $I_s$ (such as better propellants, more favorable nozzle area ratio, or higher chamber pressure) reflects itself in improved vehicle performance, provided that such an improvement does not also cause an excessive increase in rocket propulsion system inert mass, which causes a decrease in the effective propellant fraction.

### 4.2. FORCES ACTING ON A VEHICLE IN THE ATMOSPHERE

The external forces commonly acting on vehicles flying in the earth's atmosphere are thrust, aerodynamic forces, and gravitational attractions. Other forces, such as wind or solar radiation pressure, are usually small and generally can be neglected for many simple calculations.

The **thrust** is the force produced by the power plant, such as a propeller or a rocket. It usually acts in the direction of the axis of the power plant, that is, along the propeller shaft axis or the rocket nozzle axis. The thrust force of a rocket with constant mass flow has been expressed by Eq. 2–6 as a function of the effective exhaust velocity $c$ and the propellant flow rate $m$. In many rockets the mass rate of propellant consumption $\dot{m}$ is essentially constant, and the starting and stopping transients are usually very short and can be neglected. Therefore, the thrust is defined by Eq. 2–13 and is restated here with different parameters:

$$F = c m = c m_p / I_s$$ \hspace{1cm} (4–9)

As explained in Chapter 3, for a given propellant the value of the effective exhaust velocity $c$ or specific impulse $I_s$ depends in part on the nozzle area.
ratio and the altitude. The value of \( c \) can increase by a relatively small factor of between 1.2 and 1.6 as altitude is increased with a maximum value in space (vacuum).

The drag \( D \) is the aerodynamic force in a direction opposite to the flight path due to the resistance of the body to motion in a fluid. The lift \( L \) is the aerodynamic force acting in a direction normal to the flight path. They are expressed as functions of the flight speed \( u \), the mass density of the fluid (air) in which the vehicle moves \( \rho \), and a typical surface area \( A \):

\[
\begin{align*}
L &= C_L \frac{1}{2} \rho Au^2 \\
D &= C_D \frac{1}{2} \rho Au^2
\end{align*}
\]

where \( C_L \) and \( C_D \) are lift and drag coefficients, respectively. For airplanes and winged missiles the area \( A \) is the wing area. For wingless missiles or space launch vehicles it is the maximum cross-sectional area normal to the missile axis. The lift and drag coefficients are primarily functions of the vehicle configuration, flight Mach number, and angle of attack, which is the angle between the vehicle axis (or the wing plane) and the flight direction. For low flight speeds the effect of Mach number may be neglected, and the drag and lift coefficients are functions of the angle of attack. A typical variation of the drag and lift coefficients for a typical supersonic missile is shown in Fig. 4–3. The values of these coefficients reach a maximum near a Mach number of unity. For wingless vehicles the angle of attack \( \alpha \) is usually very small (\( 0 < \alpha < 1^\circ \)). The density and other properties of the atmosphere are listed in Appendix 2. The local density of the earth's atmosphere can vary day by day by a factor up to 2 (for altitudes of 300 to 1200 km) depending on solar activity and night-to-day temperature variations. This introduces a major unknown in the drag. The aerodynamic forces are affected by the flow and pressure distribution of the rocket exhaust gases, as explained in Chapter 20.

The flight regime in the neighborhood of Mach 1 is called the transonic phase of flight. Here strong unsteady aerodynamic forces can develop (due to buffeting) which are reflected in the steep rise and decrease of the coefficients seen in Fig. 4–3. Marginal transonic load capabilities have led to structural failures of flight vehicles.

For space launch vehicles and ballistic missiles the integrated drag loss, when expressed in terms of \( \Delta u \), is typically 5 to 10% of the final ideal vehicle velocity. This relatively low value is due to the fact that the air density is low at high altitudes, when the velocity is high, and at low altitudes the air density is high but the flight velocity and thus the dynamic pressure are low.

Gravitational attraction is exerted upon a flying space vehicle by all planets, stars, the moon, and the sun. Gravity forces pull the vehicle in the direction of the center of mass of the attracting body. Within the immediate vicinity of the earth, the attraction of other planets and bodies is negligibly small compared to the earth's gravitational force. This force is the weight.

\[
g = g_0 \left( \frac{R_0}{R} \right)^2 = g_0 \left( \frac{R_0}{R_0 + h} \right)^2
\]

(4–12)
where \( h \) is the altitude. At the equator the spherical earth’s radius is 6378.388 km and the standard value of \( g_0 \) is 9.80665 m/sec\(^2\). At a distance as far away as the moon, the earth’s gravity acceleration is only about \( 3.3 \times 10^{-4} g_0 \). In a more accurate analysis the value of \( g \) will vary locally with the earth’s bulge at the equator, the high mountains, and the difference of densities of specific earth regions.

### 4.3. Basic Relations of Motion

For a vehicle that flies within the proximity of the earth, the gravitational attraction of all other heavenly bodies may usually be neglected. Let it be assumed that the vehicle is moving in rectilinear equilibrium flight and that all control forces, lateral forces, and moments that tend to turn the vehicle are zero. The trajectory is two dimensional and is contained in a fixed plane. The vehicle has wings that are inclined to the flight path at an angle of attack \( \alpha \) and that give a lift in a direction normal to the flight path. The direction of flight does not necessarily coincide with the direction of thrust. Figure 4–4 shows these conditions schematically.

Let \( \psi \) be the angle of the flight path with the horizontal and \( \theta \) the angle of the direction of thrust with the horizontal. In the direction of the flight path the product of the mass and the acceleration has to equal the sum of all forces, namely the propulsive, aerodynamic, and gravitational forces:

\[
    m \left( \frac{du}{dt} \right) = F \cos(\psi - \theta) - D - mg \sin \theta \tag{4–13}
\]

The acceleration perpendicular to the flight path is \( u(d\theta/dt) \); for a constant value of \( u \) and the instantaneous radius \( R \) of the flight path it is \( u^2/R \). The equation of motion in a direction normal to the flight velocity is

\[
    mu(d\theta/dt) = F \sin(\psi - \theta) + L - mg \cos \theta \tag{4–14}
\]

By substituting from Eqs. 4–10 and 4–11, these two basic equations can be solved for the accelerations as

\[
    \frac{du}{dt} = \frac{F}{m} \cos(\psi - \theta) - \frac{C_D}{2m} \rho u^2 A - g \sin \theta \tag{4–15}
\]

\[
    \frac{d\theta}{dt} = \frac{F}{m} \sin(\psi - \theta) + \frac{C_L}{2m} \rho u^2 A - g \cos \theta \tag{4–16}
\]

No general solution can be given to these equations, since \( t, u, C_D, C_L, \rho, \theta, \) or \( \psi \) can vary independently with time, mission profile, or altitude. Also, \( C_D \) and \( C_L \) are functions of velocity or Mach number. In a more sophisticated analysis other factors may be considered, such as the propellant used for nonpropulsive purposes (e.g., attitude control or flight stability). See Refs. 4–1, 4–8, 4–11, and 4–12 for a background of flight performance in some of the flight regimes. Different flight performance parameters are maximized or optimized for different rocket flight missions or flight regimes, such as \( \Delta u \), range, orbit height and shape, time-to-target, or altitude. Rocket propulsion systems are usually tailored to fit specific flight missions.

Equations 4–15 and 4–16 are general and can be further simplified for various special applications, as shown in subsequent sections. Results of such iterative calculations of velocity, altitude, or range using the above two basic equations often are adequate for rough design estimates. For actual trajectory analyses, navigation computation, space flight path determination, or missile-firing tables, this two-dimensional simplified theory does not permit sufficiently accurate results. The perturbation effects, such as those listed in Section 4.4, must then be considered in addition to drag and gravity, and digital computers are necessary to handle the complex relations. An arbitrary division of the trajectory into small elements and a step-by-step or numerical integration to define a trajectory are usually indicated. The more generalized three-body theory includes the gravitational attraction among three masses (for example, the earth, the moon, and the space vehicle) and is considered necessary for many space flight problems (see Refs. 4–2 to 4–5). When the propellant flow and the thrust are not constant, when the flight path is three dimensional, the form and the solution to the equations above become more complex.

A form of Eqs. 4–15 and 4–16 can also be used to determine the actual thrust or actual specific impulse during actual vehicle flights from accurately observed trajectory data, such as from optical or radar tracking data. The vehicle acceleration \((du/dt)\) is essentially proportional to the net thrust and, by making an assumption or measurement on the propellant flow (which usually varies in a predetermined manner) and an analysis of aerodynamic forces, it is possible to determine the rocket propulsion system’s actual thrust under flight conditions.

For each mission of flight one can obtain actual histories of velocities and distances traveled and thus complete trajectories when integrating Eqs. 4–15 and 4–16. The more general case requires six equations; three for translation along each of three perpendicular axes and three for rotation about these axes. The
Some are two dimensional, relatively simple, and used for making preliminary estimates or comparisons of alternative flight paths, alternative vehicle designs, or alternative propulsion schemes. Several use a stationary flat earth, while others use a rotating curved earth. Three-dimensional programs also exist, are used for more accurate flight path analyses, and include some or all significant perturbations, orbit plane changes, or flying at angles of attack. As explained in Ref. 4–4, they are more complex.

If the flight trajectory is vertical (as for a sounding rocket), then \( \sin \theta = 1.0 \) and \( \cos \theta = 0 \). Equation 4-17 can be modified:

\[
\frac{du}{dt} = \frac{c \xi / t_p}{1 - \xi / t_p} - g - \frac{C_D \frac{1}{2} \rho u^2 A / m_0}{1 - \xi / t_p} \tag{4-18}
\]

The velocity at the end of burning can be found by integrating between the limits of \( t = 0 \) and \( t = t_p \) when \( u = u_0 \) and \( u = u_p \). The first two terms can readily be integrated. The last term is of significance only if the vehicle spends a considerable portion of its time within the atmosphere. It can be integrated graphically or by numerical methods, and its value can be designated as \( B C_D A / m_0 \) such that

\[
B = \int_0^{t_p} \frac{\frac{1}{2} \rho u^2}{1 - \xi / t_p} \, dt
\]

The cutoff velocity or velocity at the end of propellant burning \( u_p \) is then

\[
u_p = -\bar{c} \ln(1 - \xi) - \bar{g} t_p - \frac{B C_D A}{m_0} + u_0 \tag{4-19}\]

where \( u_0 \) is the initial velocity, such as may be given by a booster, \( \bar{g} \) is an average gravitational attraction evaluated with respect to time and altitude from Eq. 4–12, and \( \bar{c} \) is a time average of the effective exhaust velocity, which is a function of altitude.

There are always a number of trade-offs in selecting the best trajectory for a rocket projectile. For example, for a fixed thrust there is a trade-off between burning time, drag, payload, maximum velocity, and maximum altitude (or range). Reference 4–2 describes the trade-offs between payload, maximum altitude, and flight stability for a sounding rocket.

If aerodynamic forces outside the earth’s atmosphere are neglected (operate in a vacuum) and no booster or means for attaining an initial velocity \( u_0 = 0 \) is assumed, the velocity at the end of the burning reached in a vertically ascending trajectory will be

\[
u_p = -\bar{c} \ln(1 - \xi) - \bar{g} t_p
= -\bar{c} \ln(MR) - \bar{g} t_p
= \bar{c} \ln(1/\text{MR}) - \bar{g} t_p \tag{4-20}\]
The first term on the right side is usually the largest and is identical to Eq. 4–6. It is directly proportional to the effective rocket exhaust velocity and is very sensitive to changes in the mass ratio. The second term is related to the earth’s gravity and is always negative during ascent, but its magnitude is small if the burning time \( t_p \) is short or if the flight takes place in high orbits or in space where \( g \) is comparatively small.

For a flight that is not following a vertical path, the gravity loss is a function of the angle between the flight direction and the local horizontal; more specifically, the gravity loss is the integral of \( g \sin \theta \, dt \).

For the simplified two-dimensional case the net acceleration \( a \) for vertical takeoff at sea level is

\[
a = \frac{F_0 g_0}{w_0} - g_0
\]

where \( a/g_0 \) is the initial takeoff acceleration in multiples of the sea-level gravitational acceleration \( g_0 \), and \( F_0/w_0 \) is the thrust-to-weight ratio at takeoff. For large surface-launched vehicles, this initial-thrust-to-initial-weight ratio typically has values between 1.2 and 2.2; for small missiles (air-to-air, air-to-surface, and surface-to-air types) this ratio is usually larger, sometimes even as high as 50 or 100. The final or terminal acceleration \( a_f \) of a vehicle in vertical ascent usually occurs just before the rocket engine is shut off and/or before the propellant is completely consumed. If drag is neglected, then

\[
a_f/g_0 = \frac{F_f/w_f}{g_0} - 1
\]

The second version of this equation applies if the powered flight path traverses a substantial change in altitude and thus a change in the value of \( g \). In a gravity-free environment this equation becomes \( a_f/g_0 = F_f/w_f \). In rockets with constant propellant flow the final acceleration is usually also the maximum acceleration, because the vehicle mass to be accelerated has its minimum value just before propellant exhaustion, and for ascending rockets the thrust usually increases with altitude. If this terminal acceleration is too large (and causes overstressing of the structure, thus necessitating an increase in structure mass), then the thrust can be designed to a lower value for the last portion of the burning period. In manned flights the maximum acceleration is limited to the maximum \( g \) loading that can be withstood by the crew.

**Example 4–1.** A simple single-stage rocket for a rescue flare has the following characteristics. Its flight path nomenclature is shown in the accompanying sketch.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch weight</td>
<td>4.0 lbf</td>
</tr>
<tr>
<td>Useful propellant weight</td>
<td>0.4 lbf</td>
</tr>
<tr>
<td>Effective specific impulse</td>
<td>120 sec</td>
</tr>
<tr>
<td>Burn time ( t_p ) (constant thrust)</td>
<td>80°</td>
</tr>
<tr>
<td>Burn time ( t_p ) (constant thrust)</td>
<td>1.0 sec</td>
</tr>
</tbody>
</table>

The heavy line in the ascending trajectory designates the powered portion of the flight.

Drag may be neglected since the flight velocities are low. Assume that the acceleration of gravity is unchanged from its sea-level value \( g_0 \), which then makes the propellant mass numerically equal to the propellant weight in the EE system, or 0.4 lbm. Also assume that start and stop transients are short and can be ignored.

Solve for the initial and final acceleration of powered flight, the maximum trajectory height and the time to reach maximum height, the range or horizontal distance to impact, and the angle at propulsion cutoff.

**SOLUTION.** We divide the flight path into three portions: the powered flight for 1 sec, the unpowered ascent after cutoff, and the free-fall descent. The thrust is obtained from Eq. 2–5:

\[
F = \frac{L_i}{t_p} = 120 \times 0.4/1.0 = 48 \text{ lbf}
\]

The initial accelerations along the \( x \) and \( y \) horizontal and vertical directions are, from Eq. 4–22,

\[
\begin{align*}
(a_0)_y &= g_0(F \sin \theta/w) - 1 = 32.17(48/4.0 \sin 80° - 1) = 348 \text{ ft/sec}^2 \\
(a_0)_x &= g_0(F \cos \theta/w) = 32.17(48/4.0 \cos 80°) = 67.03 \text{ ft/sec}^2
\end{align*}
\]

At thrust termination the initial flight acceleration becomes

\[
a_0 = \sqrt{(a_0)_y^2 + (a_0)_x^2} = 354.4 \text{ ft/sec}^2
\]

The vertical and horizontal components of the velocity \( u_p \) at the end of powered flight are obtained from Eq. 4–20. Note that the vehicle mass has been diminished by the propellant that has been consumed:

\[
\begin{align*}
(u_p)_y &= c \ln\left(\frac{m_0}{m_f}\right) \sin \theta - g_0 \frac{u_p}{c} = 32.17 \times 120 \times \ln(4/3.6) \times 0.984 - 32.17 \times 1.0 \\
&= 368 \text{ m/sec} \\
(u_p)_x &= c \ln\left(\frac{m_0}{m_f}\right) \cos \theta = 32.17 \times 120 \times \ln(4/3.6) \times 0.1736 = 70.6 \text{ m/sec}
\end{align*}
\]
The trajectory angle with the horizontal at rocket cutoff for a dragless flight is

\[
\tan^{-1}\left(\frac{368}{70.6}\right) = 79.1^\circ
\]

The final acceleration is found, using Eq. 4–22 with the final mass, as

\[
a_f = 400 \text{ m/sec}^2.
\]

For the powered flight, the coordinates at propulsion burnout \(y_p\) and \(x_p\) can be calculated from the time integration of their respective velocities. The results are

\[
y_p = c_f(1 - \ln(m_0/m_f)/(m_0/m_f - 1)) \sin \theta - \frac{1}{2}gt^2_p = 32.17120(1 - \ln(4/3.6)/(4/3.6 - 1)) \times 0.984 - \frac{1}{2} \times 32.17 = (1.0)^2 = 181 \text{ ft}
\]

\[
x_p = c_f(1 - \ln(m_0/m_f)/(m_0/m_f - 1)) \cos \theta = 32.17120(1 - \ln(4/3.6)/(4/3.6 - 1)) \times 0.173 = 94.7 \text{ ft}
\]

The unpowered part of the trajectory reaches zero vertical velocity at its zenith. The height gained in unpowered free flight may be obtained by equating the vertical kinetic energy at power cutoff to its equivalent potential energy,

\[
g_0(y_z - y_p) = \frac{1}{2}(u_p)^2
\]

so that

\[
y_z - y_p = \frac{1}{2}(u_p)^2/g_0 = \frac{1}{2}(368)^2/32.17 = 2105 \text{ ft}
\]

The maximum height or zenith location thus becomes \(y_z = 2105 + 181 = 2259 \text{ ft}\).

What remains now is to solve the free-flight portion of vertical descent. The time for descent from the zenith is \(t_z = \sqrt{2y_z/g_0} = 11.85 \text{ sec}\) and the final vertical or impact vertical velocity \((u_p)_z = g_0 t_z = 381 \text{ ft/sec}\).

The total horizontal range to the zenith is the sum of the powered and free-flight contributions. During free flight the horizontal velocity remains unchanged at 70.6 ft/sec because there are no accelerations (i.e., no drag, wind, or gravity component). We now need to find the free-flight time from burnout to the zenith, which is \(t = (u_p)_y/g_0 = 11.4 \text{ sec}\). The total free-flight time becomes \(t_f = 11.4 + 11.85 = 23.25 \text{ sec}\).

Now, the horizontal or total range becomes \(\Delta x = 34.7 + 70.6 \times 23.25 = 1676 \text{ ft}\).

The impact angle would be around 79°. If drag had been included, solving this problem would have required information on the drag coefficient \(C_D\) and a numerical solution using Eq. 4–18. All resulting velocities and distances would turn out somewhat lower in value. A set of flight trajectories for sounding rockets has been worked out in Ref. 4–3.

### 4.4. SPACE FLIGHT

Newton’s law of gravitation defines the attraction of gravitational force \(F_g\) between two bodies in space as follows:

\[
F_g = Gm_1m_2/R^2 = \mu m_2/R^2
\]

Here \(G\) is the universal gravity constant \((G = 6.67 \times 10^{-11} \text{m}^3/\text{kg} \cdot \text{sec}^2)\), \(m_1\) and \(m_2\) are the masses of the two attracting bodies (such as the earth and the moon, the earth and a spacecraft, or the sun and a planet), and \(R\) is the distance between their centers of mass. The earth’s gravitational constant \(\mu\) is the product of Newton’s universal constant \(G\) and the mass of the earth, \(m_1(5.974 \times 10^{24} \text{kg})\). It is \(\mu = 3.98600 \times 10^{14} \text{m}^3/\text{sec}^2\).

The rocket offers a means for escaping the earth for lunar and interplanetary travel, for escaping our solar system, and for creating a stationary or moving station in space. The flight velocity required to escape from the earth can be found by equating the kinetic energy of a moving body to the work necessary to overcome gravity, neglecting the rotation of the earth and the attraction of other celestial bodies:

\[
\frac{1}{2}mv^2 = m \int g \, dR
\]

By substituting for \(g\) from Eq. 4–12 and by neglecting air friction the following relation for the escape velocity \(v_e\) is obtained:

\[
v_e = R_0 \sqrt{\frac{2\mu}{R_0 + h}} = \sqrt{\frac{2\mu}{R}} (4–25)
\]

Here \(R_0\) is the effective earth mean radius (6374.2 km), \(h\) is the orbit altitude above sea level, and \(g_0\) is the acceleration of gravity at the earth’s surface (9.806 m/sec). The satellite flight radius \(R\) measured from the earth’s center is \(R = R_0 + h\). The velocity of escape at the earth’s surface is 11,179 m/sec or 36,676 ft/sec and does not vary appreciably within the earth’s atmosphere, as shown by Fig. 4–6. Escape velocities for surface launch as given in Table 4–1 for the sun, the planets, and the moon. Launching from the earth’s surface at escape velocity is not practical. As a vehicle ascends through the earth’s atmosphere, it is subject to severe aerodynamic heating and dynamic pressures. A practical launch vehicle has to traverse the atmosphere at relatively low velocity and accelerate to the high velocities beyond the dense atmosphere. For example, during a portion of the Space Shuttle’s ascent, its main engines are actually throttled to a lower thrust to avoid excessive pressure and heating. Alternatively, an escape vehicle can be launched from an orbiting space station or from an orbiting Space Shuttle.

A rocket spaceship can become a satellite of the earth and revolve around the earth in a fashion similar to that of the moon. Satellite orbits are usually elliptical and some are circular. Low earth orbits, typically below 500 km altitude, are designated by the letters LEO. The altitude of the orbit is usually above the earth’s atmosphere, because this minimizes the expanding of energy to overcome the small drag which pulls the vehicle closer to the earth. The effects of the radiation in the Van Allen belt on human beings and sensitive equipment sometimes necessitate the selection of an earth orbit at low altitude.

For a circular trajectory the velocity of a satellite must be sufficiently high so that its centripetal force balances the earth’s gravitational attraction:

\[
mv^2/R = mg
\]
FIGURE 4-6. Orbital energy, orbital velocity, period of revolution, and earth escape velocity of a space vehicle as a function of altitude for circular satellite orbits. It is based on a spherical earth and neglects the earth’s oblate shape, rotation, and atmospheric drag.

For a circular orbit, the satellite velocity $u_s$ is found by using Eq. 4-12,

$$u_s = \sqrt{g_0 R_0/(R_0 + h)} = \sqrt{\mu/R}$$

(4-26)

which is smaller than the escape velocity by a factor of $\sqrt{2}$. The period $\tau$ in seconds of one revolution for a circular orbit relative to a stationary earth is

$$\tau = 2\pi(R_0 + h)/u_s = 2\pi(R_0 + h)^{3/2}/(R_0\sqrt{g_0})$$

(4-27)

The energy $E$ necessary to bring a unit of mass into a circular satellite orbit neglecting drag consists of kinetic and potential energy, namely,

$$E = \frac{1}{2}u_s^2 + \int_{R_0}^{R} g dR$$

$$= \frac{1}{2} \frac{R_0^2}{R_0 + h} \frac{g_0}{R_0} + \int_{R_0}^{R} \frac{R_0^2}{R_0 + h} dR = \frac{1}{2} \frac{R_0}{R_0 + h} \frac{R_0 + 2h}{R_0}$$

(4-28)

The escape velocity, satellite velocity, satellite period, and satellite orbital energy are shown as functions of altitude in Fig. 4-6.

A satellite circulating around the earth at an altitude of 300 miles or 482.8 km has a velocity of about 7375 m/sec or 24,200 ft/sec, circles a stationary earth in 1.63 hr, and ideally requires an energy of $3.35 \times 10^7$ J to place 1 kg of spaceship mass into its orbit. An equatorial satellite in a circular orbit at an altitude of

<table>
<thead>
<tr>
<th>Name</th>
<th>Mean</th>
<th>Moon</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
<th>Uranus</th>
<th>Neptune</th>
<th>Pluto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Radius (km)</td>
<td>1,393,000</td>
<td>332,950</td>
<td>6,796</td>
<td>10,820</td>
<td>12,104</td>
<td>14,298</td>
<td>18,033</td>
<td>24,622</td>
<td>28,672</td>
<td>44,450</td>
<td>58,950</td>
</tr>
<tr>
<td>Revolution</td>
<td>0.883</td>
<td>27.3 days</td>
<td>247.97 days</td>
<td>24.7 days</td>
<td>12.74 days</td>
<td>6.70 days</td>
<td>11.88 days</td>
<td>24.62 days</td>
<td>9.52 days</td>
<td>16.48 days</td>
<td>22.67 days</td>
</tr>
<tr>
<td>Period of Orbit (days)</td>
<td>1.34</td>
<td>1.00</td>
<td>0.66</td>
<td>0.63</td>
<td>0.53</td>
<td>0.52</td>
<td>0.55</td>
<td>0.60</td>
<td>0.69</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>Escape Velocity at Surface (m/sec)</td>
<td>272.4</td>
<td>71.8</td>
<td>5.5</td>
<td>6.7</td>
<td>9.3</td>
<td>11.8</td>
<td>14.9</td>
<td>20.0</td>
<td>26.0</td>
<td>29.6</td>
<td>37.2</td>
</tr>
<tr>
<td>Acceleration of Gravity at Surface (m/sec$^2$)</td>
<td>0.4</td>
<td>0.16</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: Data in part from Refs. 4-3 and 4-4.
6.611 earth radii (about 26,200 miles, 42,200 km, or 22,700 nautical miles) has a period of revolution of exactly 24 hr. It will appear stationary to an observer on earth. This is known as a synchronous satellite in geosynchronous earth orbit, usually abbreviated as GEO. It is used extensively for communications satellite applications. In Section 4.7 on launch vehicles we will describe how the payload of a given space vehicle diminishes as the orbit circular altitude is increased and as the inclination (angle between orbit plane and earth equatorial plane) is changed. See Refs. 4–3, 4–4, 4–5, 4–6, and 4–9.

Elliptical Orbits

The circular orbit described above is a special case of the more general elliptic orbit shown in Fig. 4–7; here the earth (or any other heavenly body around which another body is moving) is located at one of the focal points of this ellipse. The equations of motion may be derived from Kepler’s laws, and the elliptical orbit can be described as follows, when expressed in polar coordinates:

\[ u = \left[ \frac{\mu}{R} \left( \frac{1}{R} - \frac{1}{a} \right) \right]^{1/2} \]  

(4–29)

where \( u \) is the velocity of the body in the elliptical orbit, \( R \) is the instantaneous radius from the center of the attracting body (a vector quantity which changes direction as well as magnitude), \( a \) is the major axis of the ellipse, and \( \mu \) is the earth’s gravitational constant with a value of \( 3.986 \times 10^{14} \text{ m}^3/\text{sec}^2 \). The symbols are defined in Fig. 4–7. From this equation it can be seen that the velocity \( u_p \) is a maximum when the moving body comes closest to its focal point at the orbit’s perigee and that its velocity \( u_a \) is a minimum at its apogee. By substituting for \( R \) in Eq. 4–29, and by defining the ellipse’s shape factor \( e \) as the eccentricity of the ellipse, \( e = \sqrt{a^2 - b^2} / a \), the apogee and perigee velocities can be expressed as

\[ u_a = \frac{\mu(1 - e)}{a(1 + e)} \]  

(4–30)

\[ u_p = \frac{\mu(1 + e)}{a(1 - e)} \]  

(4–31)

Another property of an elliptical orbit is that the product of velocity and instantaneous radius remains constant for any location \( x \) or \( y \) on the ellipse, namely, \( u_a R_a = u_p R_p = uR \). The exact path that a satellite takes depends on the velocity (magnitude and vector orientation) with which it is started or injected into its orbit.

For interplanetary transfers the ideal mission can be achieved with minimum energy in a simple transfer ellipse, as suggested originally by Hohmann (see Ref. 4–6). Assuming the planetary orbits about the sun to be circular and coplanar, it can be demonstrated that the path of minimum energy is an ellipse tangent to the planetary orbits as shown in Fig. 4–8. This operation requires a velocity increment (relatively high thrust) at the initiation (planet A at \( t_1 \)) and another at termination (planet B at \( t_2 \)): both increments are the velocity differences between the respective circular planetary velocities and the perigee and apogee velocities which define the transfer ellipse. The thrust levels at the beginning and end maneuvers of the Hohmann ellipse must be high enough to give a short operating time and the acceleration of at least 0.01 \( g \), but preferably more. With electrical propulsion these acceleration would be about \( 10^{-5} \) \( g \), the operating time would be weeks or months, and the best transfer trajectories would be very different from a Hohmann ellipse; they are described in Chapter 17.

**FIGURE 4–7.** Elliptical orbit; the attracting body is at one of the focal points of the ellipse.

**FIGURE 4–8.** Schematic diagram of interplanetary transfer paths. These same transfer maneuvers apply when going from a low-altitude earth satellite orbit to a higher orbit.
The departure date or the relative positions of the launch planet and the target planet for a planetary transfer mission are critical, because the spacecraft has to meet with the target planet when it arrives at the target orbit. The transfer time \((t_2 - t_1)\) for a Hohmann ellipse flight starting on earth is about 116 hr to go to the moon and about 259 days to Mars. If a faster orbit (shorter transfer time) is desired (see dashed lines in Fig. 4–8), it requires more energy than a Hohmann transfer ellipse. This means a larger vehicle with more propellant and a larger propulsion system or more total impulse. There also is a time window for a launch of a spacecraft that will make a successful rendezvous. For a Mars mission an earth-launched spacecraft may have a launch time window of more than two months. A Hohmann transfer ellipse or a faster transfer path applies not only to planetary flight but also to earth satellites, when an earth satellite goes from one circular orbit to another (but within the same plane). Also, if one spacecraft goes to a rendezvous with another spacecraft in a different orbit, the two spacecraft have to be in the proper predetermined positions prior to the launch to simultaneously reach their rendezvous location.

When the launch orbit (or launch planet) is not in the same plane as the target orbit, then additional energy will be needed by applying thrust in a direction normal to the launch orbit plane. More information is in Refs. 4–3, 4–4, 4–6, and 4–10.

**Example 4–2.** A satellite is launched from a circular equatorial parking orbit at an altitude of 160 km into a coplanar circular synchronous orbit by using a Hohmann transfer ellipse. Assume a homogeneous spherical earth with a radius of 6371 km. Determine the velocity increments for entering the transfer ellipse and for achieving the synchronous orbit at 42,200 km altitude. See Fig. 4–8 for the terminology of the orbits.

**SOLUTION.** The orbits are \(R_A = 6.531 \times 10^6 \text{ m}; R_B = 48.751 \times 10^6 \text{ m}\). The major axis \(a\) of the transfer ellipse is

\[
a = \frac{1}{2} (R_A + R_B) = 27.551 \times 10^6 \text{ m}
\]

The orbit velocities of the two satellites in the two orbits are

\[
u_A = \sqrt{\mu/R_A} = [3.986005 \times 10^{14}/6.571 \times 10^6]^{1/2} = 7788 \text{ m/sec}
\]

\[
u_B = \sqrt{\mu/R_B} = 2864.7 \text{ m/sec}
\]

The velocities needed to enter and exit the transfer ellipse are

\[
(\nu_{Ae}) A = \sqrt{\mu(2/R_A) - (1/a)}^{1/2} = 10,337 \text{ m/sec}
\]

\[
(\nu_{Be}) B = \sqrt{\mu(2/R_B) - (1/a)}^{1/2} = 1394 \text{ m/sec}
\]

The changes in velocity going from parking orbit to ellipse and from ellipse to final orbit are

\[
\Delta u_A = |(\nu_{Be}) A - \nu_A| = 2549 \text{ m/sec}
\]

\[
\Delta u_B = |\nu_B - (\nu_{Be}) B| = 1471 \text{ m/sec}
\]

**FIGURE 4–9.** Long-range ballistic missiles follow an elliptical free flight trajectory, which is drag free, with the earth's center as one of the focal points. The surface launch is usually vertically up (not shown here) but the flight path is quickly tilted during the early powered flight to enter into an elliptic trajectory. The ballistic range is the arc distance on the earth's surface. The same elliptical flight path can be used by launch vehicles for satellites; another powered flight period occurs (called orbit injection) just as the vehicle is at its elliptical apogee (as indicated by the arrow), causing the vehicle to enter an orbit.

The total velocity change for the transfer maneuvers is

\[
\Delta u_{total} = \Delta u_A + \Delta u_B = 4020 \text{ m/sec}
\]

Figure 4–9 shows the elliptical transfer trajectory of a ballistic missile or a satellite asent vehicle. During the initial powered flight the trajectory angle is adjusted by the guidance system to an angle that will allow the vehicle to reach the apogee of its elliptical path exactly at the desired orbit altitude. For the ideal satellite the simplified theory assumes that orbit injection is an essentially instantaneous application of the total impulse as the ballistic trajectory reaches its apogee or zenith. In reality the rocket propulsion system operates over a finite time, during which gravity losses and changes in altitude occur.

**Deep Space**

Lunar and interplanetary missions include circumnavigation, landing, and return flights to the moon, Venus, Mars, and other planets. The energy necessary to
escape from the earth can be calculated as $\frac{1}{2}mv_\text{e}^2$ from Eq. 4–25. It is $6.26 \times 10^7$ J/kg, which is more than that required for a satellite. The gravitational attraction of various heavenly bodies and their respective escape velocities depends on their masses and diameters; approximate values are listed in Table 4–1. An idealized diagram of an interplanetary landing mission is shown in Fig. 4–10.

The escape from the solar system requires approximately $5.03 \times 10^8$ J/kg. This is eight times as much energy as is required for escape from the earth. There is technology to send small, unmanned probes away from the sun to outer space; as yet there needs to be an invention and demonstrated proof of a long-duration, novel, rocket propulsion system before a mission to the nearest star can be achieved. The trajectory for a spacecraft to escape from the sun is either a parabola (minimum energy) or a hyperbola. See Refs. 4–6 and 4–10.

**Perturbations**

This section gives a brief discussion of the disturbing torques and forces which cause perturbations or deviations from any intended space flight path or any satellite’s flight orbit. For a more detailed treatment of flight paths and their perturbations, see Refs. 4–3, 4–4, and 4–13. A system is needed to measure the satellite’s position and deviation from the intended flight path, to determine the needed periodic correction maneuver, and then to counteract, control, and correct them. This is called orbit maintenance; it corrects the perturbed or altered orbit by periodically applying small rocket propulsion forces in predetermined directions. Typically, the corrections are performed by a set of small reaction control thrusters which provide predetermined total impulses into the desired directions. These corrections are needed throughout the life of the spacecraft (for 1 to 20 years or sometimes more) to overcome the effects of the disturbances and maintain the intended flight regime.

Perturbations can be categorized as short term and long term. The daily or orbital period oscillating forces are called **diurnal** and those with long periods are called **secular**.

High-altitude earth satellites (36,000 km and higher) experience perturbing forces primarily as gravitational pull from the sun and the moon, with the forces acting in different directions as the satellite flies around the earth. This third-body effect can increase or decrease the velocity magnitude and change its direction. In extreme cases the satellite can come very close to the third body, such as a planet or one of its moons, and undergo what is called a hyperbolic maneuver that will radically change the trajectory. This encounter can be used to increase or decrease the energy of the satellite and intentionally change the velocity and the shape of the orbit.

Medium- and low-altitude satellites (500 to 35,000 km) experience perturbations because of the earth’s oblateness. The earth bulges in the vicinity of the equator and a cross section through the poles is not entirely circular. Depending on the inclination of the orbital plane to the earth equator and the altitude of the satellite orbit, two perturbations result: (1) the regression of the nodes and (2) shifting of the apsidal line (major axis). Regression of the nodes is shown in Fig. 4–11 as a rotation of the plane of the orbit in space, and it can be as high as 9° per day at relatively low altitudes. Theoretically, regression does not occur in equatorial orbits.

Figure 4–12 shows an exaggerated shift of the apsidal line, with the center of the earth remaining as a focus point. This perturbation may be visualized as the
movement of the prescribed elliptical orbit in a fixed plane. Obviously, both the apogee and perigee points change in position, the rate of change being a function of the satellite altitude and plane inclination angle. At an apogee altitude of 1000 nautical miles (n.m.) and a perigee of 100 n.m. in an equatorial orbit, the apsidal drift is approximately $10^{-5}$ per day.

Satellites of modern design, with irregular shapes due to protruding antennas, solar arrays, or other asymmetrical appendages, experience torques and forces that tend to perturb the satellite’s position and orbit throughout its orbital life. The principal torques and forces result from the following factors:

1. **Aerodynamic drag.** This factor is significant at orbital altitudes below 500 km and is usually assumed to cease at 800 km above the earth. Reference 4-8 gives a detailed discussion of aerodynamic drag, which, in addition to affecting the attitude of unsymmetrical vehicles, causes a change in elliptical orbits known as apsidal drift, a decrease in the major axis, and a decrease in eccentricity of orbits about the earth. See Refs. 4-6, 8-12, and 13.

2. **Solar radiation.** This factor dominates at high altitudes (above 800 km) and is due to impingement of solar photons upon satellite surfaces. The solar radiation pressure $p$ (N/m$^2$) on a given surface of the satellite in the vicinity of the earth exposed to the sun can be determined as

$$p = 4.5 \times 10^{-6} \cos \theta [(1 \times k_s) \cos \theta + 0.67 k_d]$$  \hspace{1cm} (4-32)

where $\theta$ is the angle (degrees) between the incident radiation vector and the normal to the surface and $k_s$ and $k_d$ are the specular and diffuse coefficients of reflectivity. Typical values are 0.9 and 0.5, respectively, for $k_s$ and $k_d$ on the body and antenna and 0.25 and 0.01, respectively, for $k_s$ and $k_d$ with solar array surfaces. The radiation intensity varies as the square of the distance from the sun (see Refs. 4-4 and 4-14). The torque $T$ on the vehicle is given by $T = pA$, where $A$ is the projected area and $l$ is the offset distance between the spacecraft’s center of gravity and the center of solar pressure. For a nonsymmetrical satellite with a large solar panel on one side the radiation will cause a small torque, which will rotate the vehicle.

3. **Gravity gradients.** Gravitational torque in spacecraft results from a variation in the gravitational force on the distributed mass of a spacecraft. Determination of this torque requires knowledge of the gravitational field and the distribution of spacecraft mass. This torque decreases as a function of the orbit radius and increases with the offset distances of masses within the spacecraft (including booms and appendages); it is most significant in large spacecraft or space stations operating in relatively low orbits (see Refs. 4-4 and 4-15).

4. **Magnetic field.** The earth’s magnetic field and any magnetic moment within the satellite interact to produce torque. The earth’s magnetic field precesses about the earth’s axis but is very weak (0.63 and 0.31 gauss at poles and equator, respectively). This field is continually fluctuating in direction and intensity because of magnetic storms and other influences. Since the field strength decreases with $1/R^3$ with the orbital altitude, magnetic field forces are often neglected in the preliminary analysis of satellite flight paths (see Ref. 4-16).

5. **Internal accelerations.** Deployment of solar array panels, the shifting of liquid propellant, the movement of astronauts or other masses within the satellite, or the “unloading” of reaction wheels can produce torques and forces.

6. For precise low earth orbits the oblateness of the earth (diameter at equator is slightly larger than diameter between poles), high mountains, or earth surface areas of different densities will perturb these orbits.

We can categorize satellite propulsion needs according to function as listed in Table 4-2, which shows the total impulse “budget” applicable to a typical high-altitude, elliptic orbit satellite. The control system designer often distinguishes two different kinds of station-keeping orbit corrections needed to keep the satellite in a synchronous position. The east–west correction refers to a correction that moves the point at which a satellite orbit intersects the earth’s equatorial plane in an east or west direction; it usually corrects forces caused largely by the oblateness of the earth. The north–south correction counteracts forces usually connected with the third-body effects of the sun and the moon.

In many satellite missions the gradual changes in orbit caused by perturbation forces are not of concern. However, in certain missions it is necessary to compensate for these perturbing forces and maintain the satellite in a specific orbit and in a particular position in that orbit. For example, a synchronous communications satellite in a Geo-Synchronous Earth Orbit or GEO needs to maintain its position and its orbit, so it will be able to (1) keep covering a specific area of the
TABLE 4–2. Propulsion Functions and Total Impulse Needs of a 2000-lbm Geosynchronous Satellite with a 7-Year Life

<table>
<thead>
<tr>
<th>Function</th>
<th>Total Impulse (N-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acquisition of orbit</td>
<td>20,000</td>
</tr>
<tr>
<td>Attitude control (rotation)</td>
<td>4,000</td>
</tr>
<tr>
<td>Station keeping, E–W</td>
<td>13,000</td>
</tr>
<tr>
<td>Station keeping, N–S</td>
<td>270,000</td>
</tr>
<tr>
<td>Repositioning (Δv, 200 ft/sec)</td>
<td>53,000</td>
</tr>
<tr>
<td>Control apsidal drift (third-body attraction)</td>
<td>445,000</td>
</tr>
<tr>
<td>Deorbit drift</td>
<td>12,700</td>
</tr>
<tr>
<td>Total</td>
<td>817,700</td>
</tr>
</tbody>
</table>

Mission Velocity

A convenient way to describe the magnitude of the energy requirement of a space mission is to use the concept of the mission velocity. It is the sum of all the flight velocity increments needed (in all the vehicle’s stages) to attain the mission objective even though these increments are provided by different propulsion systems. In the simplified sketch of a planetary landing mission shown in Fig. 4–10, it is the sum of all the Δv velocity increments shown by the heavy lines (rocket-powered flight segments) of the trajectories. Even through some of the velocity increments were achieved by retro-action (a negative propulsion force to decelerate the flight velocity), these maneuvers required energy and their absolute magnitude is counted in the mission velocity. The initial velocity from the earth’s rotation (464 m/sec at the equator and 408 m/sec at a launch station at 28.5° latitude) does not have to be provided by the vehicle’s propulsion systems. For example, the required mission velocity for launching at Cape Kennedy, bringing the space vehicle into an orbit at 110 km, staying in orbit for a while, and then entering a deorbit maneuver has the Δv components shown in Table 4–3.

The required mission velocity is the sum of the absolute values of all translation velocity increments that have forces going through the center of gravity of the vehicle (including turning maneuvers) during the flight of the mission. It is the theoretical hypothetical velocity that can be attained by the vehicle in a gravity-free vacuum, if all the propulsive energy of the momentum-adding thrust chambers in all stages were to be applied in the same direction. It is useful for comparing one flight vehicle design with another and as an indicator of the mission energy.

The required mission velocity has to be equal to the “supplied” mission velocity, that is, the sum of all the velocity increments provided by the propulsion systems of each of the various vehicle stages. The total velocity increment to be “supplied” by the shuttle’s propulsion systems for the shuttle mission described below (solid rocket motor strap-on boosters, main engines and, for orbit injection, also the increment from the orbital maneuvering system—all shown in Fig. 1–14) has to equal or exceed 9347 m/sec. If reaction control system propellant and an uncertainty factor are added, it will need to exceed 9621 m/sec. With chemical propulsion systems and a single stage, we can achieve a space mission velocity of 4000 to 13,000 m/sec, depending on the payload, mass ratio, vehicle design, and propellant. With two stages it can be between perhaps 12,000 and 22,000 m/sec.

Rotational maneuvers, described later, do not change the flight velocity and some analysts do not add them to the mission velocity requirements. Also, maintaining a satellite in orbit against long-term perturbing forces (see prior section) is often not counted as part of the mission velocity. However, the designers need to provide additional propulsion capability and propellants for these purposes. These are often separate propulsion systems, called reaction control systems.

Typical vehicle velocities required for various interplanetary missions have been estimated as shown in Table 4–4. By starting interplanetary journeys from
TABLE 4-4. Approximate Vehicle Mission Velocities for Typical Interplanetary Missions

<table>
<thead>
<tr>
<th>Mission</th>
<th>Ideal Velocity (km/sec)</th>
<th>Approximate Actual Velocity (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite orbit around earth (no return)</td>
<td>7.9–10</td>
<td>9.1–12.5</td>
</tr>
<tr>
<td>Escape from earth (no return)</td>
<td>11.2</td>
<td>12.9</td>
</tr>
<tr>
<td>Escape from moon</td>
<td>2.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Earth to moon (soft landing on moon, no return)</td>
<td>13.1</td>
<td>15.2</td>
</tr>
<tr>
<td>Earth to Mars (soft landing)</td>
<td>17.5</td>
<td>20</td>
</tr>
<tr>
<td>Earth to Venus (soft landing)</td>
<td>22</td>
<td>25</td>
</tr>
<tr>
<td>Earth to moon (landing on moon and return to earth(^a))</td>
<td>15.9</td>
<td>17.7</td>
</tr>
<tr>
<td>Earth to Mars (landing on Mars and return to earth(^a))</td>
<td>22.9</td>
<td>27</td>
</tr>
</tbody>
</table>

\(^a\)Assumes air braking within atmospheres.

TABLE 4-5. Approximate Relative Payload-Mission Comparison Chart for Typical High-Energy Chemical Multistage Rocket Vehicles

<table>
<thead>
<tr>
<th>Mission</th>
<th>Relative Payload(^a) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth satellite</td>
<td>100</td>
</tr>
<tr>
<td>Earth escape</td>
<td>35–45</td>
</tr>
<tr>
<td>Earth 24-hr orbit</td>
<td>10–25</td>
</tr>
<tr>
<td>Moon landing (hard)</td>
<td>35–45</td>
</tr>
<tr>
<td>Moon landing (soft)</td>
<td>10–20</td>
</tr>
<tr>
<td>Moon circumnavigation (single fly-by)</td>
<td>30–42</td>
</tr>
<tr>
<td>Moon satellite</td>
<td>20–30</td>
</tr>
<tr>
<td>Moon landing and return</td>
<td>1–4</td>
</tr>
<tr>
<td>Moon satellite and return</td>
<td>8–15</td>
</tr>
<tr>
<td>Mars flyby</td>
<td>20–30</td>
</tr>
<tr>
<td>Mars satellite</td>
<td>10–15</td>
</tr>
<tr>
<td>Mars landing</td>
<td>0.5–3</td>
</tr>
</tbody>
</table>

\(^a\)300 nautical miles (555.6 km) earth orbit is 100% reference.

a space satellite station, a considerable saving in this vehicle velocity can be achieved, namely, the velocity necessary to achieve the earth-circling satellite orbit. As the space flight objective becomes more ambitious, the mission velocity is increased. For a given single or multistage vehicle it is possible to increase the vehicle’s terminal velocity, but usually only at the expense of payload. Table 4–5 shows some typical ranges of payload values for a given multistage vehicle as a percentage of a payload for a relatively simple earth orbit. Thus a vehicle capable of putting a substantial payload into a near-earth orbit can only land a very small fraction of this payload on the moon, since it has to have additional upper stages, which displace payload mass. Therefore, much larger vehicles are required for space flights with high mission velocities if compared to a vehicle of less mission velocity but identical payload. The values listed in Tables 4–4 and 4–5 are only approximate because they depend on specific vehicle design features, the propellants used, exact knowledge of the trajectory—time relation, and other factors that are beyond the scope of this short treatment.

4.5. FLIGHT MANEUVERS

In this section we describe different flight maneuvers and relate them to specific propulsion system types. See Refs. 4–5, 4–11, 4–12, 4–13, and 4–14. The three categories of maneuvers are:

1. In translation maneuvers the rocket propulsion thrust vector goes through the center of gravity of the vehicle. The vehicle momentum is changed in the direction of the flight velocity. An example of several powered (translational maneuvers) and unpowered (coasting) segments of a complex space flight trajectory is shown in schematic, simplified form in Fig. 4–10. To date, most maneuvers have used chemical propulsion systems.

2. In truly rotational maneuvers there is no net thrust acting on the vehicle. These are true force couples that apply only torque. It requires four thrusters to be able to rotate the vehicle in either direction about any one axis (two thrusters apart, firing simultaneously, but in opposite rotational directions). These types of maneuver are usually provided by reaction control systems. Most have used multiple liquid propellant thrusters, but in recent years selected space missions have used electrical propulsion.

3. A combination of categories 1 and 2, such as a misaligned thrust vector of a large thrust chamber that does not go exactly through the center of gravity of the vehicle. The misalignment can be corrected by changing the vector direction of the main propulsion system (thrust vector control) during powered flight or by applying a simultaneous compensating torque from a separate reaction control system.

The following types of space flight maneuvers and vehicle accelerations use rocket propulsion. All propulsion operations are usually controlled (started, monitored, and stopped) by the vehicle’s guidance and control system.

\( a \). First stage, its upper stage propulsion systems and strap-on boosters add momentum during launch and ascent. They require rocket propulsion of high or medium thrusts and limited durations (typically 0.7 to 8 min). To date all have used chemical propulsion systems. They constitute the major mass of the space vehicle and are discussed further in the next section.
b. Orbit injection or transferring from one orbit to another requires accurately predetermined total impulses. It can be performed by the main propulsion system of the top stage of the launch vehicle. More often it is done by a separate propulsion system at lower thrust levels than the upper stages in item (a) above. Orbit injection can be a single thrust operation after ascent from an earth launch station. If the flight path is a Hohmann transfer ellipse (minimum energy) or a faster transfer orbit, then two thrust application periods are necessary, one at the beginning and one at the end of the transfer path. For injection into earth orbit, the thrust levels are typically between 200 and 45,000 N or 50 and 11,000 lbf, depending on the payload size, transfer time, and specific orbit. If the new orbit is higher, then the thrusts are applied in the flight direction. If the new orbit is at a lower altitude, then the thrusts must be applied in a direction opposite to the flight velocity vector. The transfer orbits can also be achieved with a very low thrust level (0.001 to 1 N) using an electric propulsion system, but the flight paths will be very different (multiloop spiral) and the transfer duration will be much longer. This is explained in Chapter 17. Similar maneuvers are also performed with lunar or interplanetary flight missions, like the planetary landing mission shown schematically in Fig. 4-10.

c. Velocity vector adjustment and minor in-flight correction maneuvers are usually performed with low-thrust, short-duration, and intermittent (pulsing) operations, using a reaction control system with multiple small liquid propellant thrusters, both for translation and rotation. The vernier rockets on a ballistic missile are used to accurately calibrate the terminal velocity vector for improved target accuracy. The reaction control system in a space launch vehicle will allow accurate orbit injection adjustment maneuvers after it is placed into orbit by another, less accurate propulsion system. Midcourse guidance-directed correction maneuvers for the trajectories of deep space vehicles fall also into this category. Propulsion systems for orbit maintenance maneuvers, also called station-keeping maneuvers (to overcome perturbing forces), keep a spacecraft in its intended orbit and orbital position and are also considered to be part of this category.

d. Reentry and landing maneuvers can take several forms. If the landing occurs on a planet that has an atmosphere, then the drag of the atmosphere will slow down the reentering vehicle. For a multiple elliptical orbit the drag will progressively reduce the periapsis altitude and the periapsis velocity on every orbit. Landing at a precise, preplanned location requires a particular velocity vector at a predetermined altitude and distance from the landing site. The vehicle has to be rotated into the right position and orientation, so as to use its heat shield correctly. The precise velocity magnitude and direction prior to entering the denser atmosphere are critical for minimizing the heat transfer (usually to the vehicle's heat shield) and to achieve touchdown at the intended landing site or, in the case of ballistic missiles, the intended target. This usually requires a relatively minor maneuver (low total impulse). If there is very little or no atmosphere (for instance, landing on the moon or Mercury), then a reverse thrust has to be applied during descent and touchdown. The rocket propulsion system usually has variable thrust to assure a soft landing and to compensate for the decrease in vehicle mass as propellant is consumed during descent. The U.S. lunar landing rocket engine, for example, had a 10 to 1 thrust variation.

e. Rendezvous and docking involve both rotational and translational maneuvers of small reaction control thrusters. Rendezvous and its time windows were discussed on page xxx. Docking (sometimes called lockon) is the linking up of two spacecraft and requires a gradual gentle approach (low thrust, pulsing node thrusters) so as not to damage the spacecraft.

f. Simple rotational maneuvers rotate the vehicle on command into a specific angular position so as to orient or point a telescope, instrument, solar panel, or antenna for purposes of observation, navigation, communication, or solar power reception. Such a maneuver is also used to keep the orientation of a satellite in a specific direction; for example, if an antenna needs to be continuously pointed at the center of the earth, then the satellite needs to be rotated around its own axis once every satellite revolution. Rotation is also used to point a nozzle of the primary propulsion system into its intended direction just prior to its start. It can also provide pulsed thrust for achieving flight stability, or for correcting angular oscillations, that would otherwise increase drag or cause tumbling of the vehicle. Spinning or rolling a vehicle about its axis will improve flight stability but will also average out the misalignment in a thrust vector. If the rotation needs to be performed quickly, then a chemical multithrust reaction control system is used. If the rotational changes can be done over a long period of time, then an electrical propulsion system with multiple thrusters is often preferred.

g. A change of plane of the flight trajectory requires the application of a thrust force (through the vehicle center of gravity) in a direction normal to the original plane of the flight path. This is usually performed by a propulsion system that has been rotated (by the reaction control system) into the proper nozzle orientation. This maneuver is done to change the plane of a satellite orbit or when going to a planet, such as Mars, whose orbit is inclined to the plane of the earth's orbit.

h. Deorbiting and disposal of used or spent spacecraft is required today to remove space debris. The spent spacecraft should not become a hazard to other spacecraft. A relatively small thrust will cause the vehicle to go to a low enough elliptical orbit so that atmospheric drag will cause further slowing. In the dense regions of the atmosphere the reentering, expended vehicle will typically break up or overheat (burn up).

i. Emergency or alternative mission. If there is a malfunction in a spacecraft and it is decided to abort the mission, such as a premature quick return to the earth without pursuing the originally intended mission, then some of the rocket engines can be used for an alternate mission. For example, the
main rocket engine in the Apollo lunar mission service module is normally used for retroaction to attain a lunar orbit and for return from lunar orbit to the earth; it can be used for emergency separation of the payload from the launch vehicle and for unusual midcourse corrections during translunar coast, enabling an emergency earth return.

Table 4–6 lists the maneuvers that have just been described, together with some others, and shows the various types of rocket propulsion system (as mentioned in Chapter 1) that have been used for each of these maneuvers. The items with a double mark “××” have been the preferred methods in recent years. The table omits several propulsion systems, such as solar thermal or nuclear rocket propulsion, because these have not yet flown in a routine space mission. The three propulsion systems on the right of Table 4–6 are electrical propulsion systems and they have very high specific impulse (see Table 2–1), which makes them very attractive for deep space missions and for certain station-keeping jobs (orbit maintenance). However, they can be applied only to missions with sufficiently long thrust action time for reaching the desired vehicle velocity or rotation positions with very small acceleration.

**Reaction Control System**

The functions of a reaction control system have been described in the previous section on flight maneuvers. They are used for the maneuvers identified by paragraphs c, e, and g. In some vehicle designs they are also used for tasks described in b, part of d, and f, if the thrust levels are low.

A reaction control system (RCS), often called an auxiliary rocket propulsion system, is needed to provide for trajectory corrections (small Δυ additions) as well as correcting the rotational or attitude position of almost all spacecraft and all major launch vehicles. If only rotational maneuvers are made, it has been called an attitude control system. The nomenclature has not been consistent throughout the industry or the literature.

An RCS can be incorporated into the payload stage and each of the stages of a multiple-stage vehicle. In some missions and designs the RCS is built into only the uppermost stage; it operates throughout the flight and provides the control torques and forces for all the stages. For large vehicle stages the thrust level of multiple thrusters of an RCS can be large (500 to 15,000 lbf) and for terminal stages in small satellites they can be small (0.01 to 10.0 lbf) and they can be pulsed as often as commanded by the vehicle flight control system. Liquid propellant rocket engines with multiple thrusters have been used for almost all launch vehicles and the majority of all spacecraft. Cold gas systems were used with early spacecraft design. In the last decade an increasing number of electrical propulsion systems have been used, primarily on spacecraft, as described in Chapter 17. The life of an RCS may be short (when used on an individual vehicle stage), or it may see use throughout the mission duration (perhaps more than 10 years) when part of an orbiting spacecraft.

---

**TABLE 4–6. Types of Rocket Propulsion System Commonly Used for Different Flight Maneuvers or Application**

<table>
<thead>
<tr>
<th>Flight Maneuvers and Applications</th>
<th>Liquid Propellant Rocket Engines</th>
<th>Solid Propellant Rocket Motors</th>
<th>Electrical Propulsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch vehicle booster</td>
<td>××</td>
<td>×</td>
<td>Arc Jet, Reasit Jet</td>
</tr>
<tr>
<td>Strap-on motor/engine</td>
<td>×</td>
<td>×</td>
<td>Ion Propulsion, Electromagnetic Propulsion</td>
</tr>
<tr>
<td>Upper stages of launch vehicle</td>
<td>××</td>
<td>××</td>
<td>Pulsed Plasma Jet</td>
</tr>
<tr>
<td>Satellite orbit injection and</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>transfer orbits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flight velocity adjustments,</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>flight path corrections,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>orbit changes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbit/position maintenance,</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>rotation of spacecraft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Docking of two spacecraft</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Reentry and landing,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>emergency maneuvers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deorbit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep space, sun escape</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Tactical missiles</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Strategic missiles</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Missile defense</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
</tbody>
</table>

Legend: × = in use; ×× = preferred for use.

The vehicle attitude has to be controlled about three mutually perpendicular axes, each with two degrees of freedom (clockwise and counterclockwise rotation), giving a total of six degrees of rotational freedom. Pitch control raises or lowers the nose of the vehicle, yaw torques induce a motion to the right or the left side, and roll torques will rotate the vehicle about its axis, either clockwise or counterclockwise. In order to apply a true torque it is necessary to use two thrust chambers of exactly equal thrust and equal start and stop times, placed an equal distance from the center of mass. Figure 4–13 shows a simple spherical...
A precise attitude angular correction can also be achieved by the use of an inertial or high-speed rotating reaction wheel, which applies torque when its rotational speed is increased or decreased. While these wheels are quite simple and effective, the total angular momentum change they can supply is generally small. By using a pair of supplementary attitude control thrust rocket units it is possible to unload or respin each wheel so it can continue to supply small angular position corrections as needed.

The torque \( T \) of a pair of thrust chambers of thrust \( F \) and a separation distance \( l \) is applied to give the vehicle with an angular or rotational moment of inertia \( M_a \) an angular acceleration of magnitude \( \alpha \):

\[
T = Fl = M_a \alpha
\]  

(4–33)

For a cylinder of equally distributed mass \( M_a = \frac{1}{2}mr^2 \) and for a homogeneous sphere it is \( M_a = \frac{2}{5}mr^2 \). The largest possible practical value of moment arm \( l \) will minimize the thrust and propellant requirements. If the angular acceleration is constant over a time period \( t \), the vehicle will move at an angular speed \( \omega \) and through a displacement angle \( \theta \), namely

\[
\omega = \alpha t \quad \text{and} \quad \theta = \frac{1}{2} \alpha t^2
\]  

(4–34)

Commonly a control system senses a small angular disturbance and then commands an appropriate correction. For this detection of an angular position change by an accurate sensor it is actually necessary for the vehicle to undergo a slight angular displacement. Care must be taken to avoid overcorrection and hunting of the vehicle position or the control system. This is one of the reasons many spacecraft require extremely short multiple pulses (0.010 to 0.030 sec) and low thrust (0.01 to 100 N) (see Refs. 4–11, 4–13, and 4–14).

Reaction control systems can be characterized by the magnitude of the total impulse, the number, thrust level, and direction of the thrusters, and their duty cycles. The duty cycle refers to the number of thrust pulses, their operating times, the times between thrust applications, and the timing of these short operations during the mission operating period. For a particular thruster, a 30% duty cycle means an average active cumulative thrust period of 30% during the propulsion system's flight duration. These propulsion parameters can be determined from the mission, the guidance and control approach, the desired accuracy, flight stability, the likely thrust misalignments of the main propulsion systems, the three-dimensional flight path variations, the perturbations to the trajectory, and several other factors. Some of these parameters are often difficult to determine.

**4.6. EFFECT OF PROPULSION SYSTEM ON VEHICLE PERFORMANCE**

This section gives several methods for improving flight vehicle performance. Most of these enhancements, listed below, are directly influenced by the flight
mission and by the selection or design of the propulsion system. A few of the flight vehicle performance improvements do not depend on the propulsion system. Most of those listed below apply to all missions, but some are peculiar to some missions only.

1. The effective exhaust velocity \( c \) and the specific impulse \( I_s \) usually have a direct effect on the vehicle’s flight performance. The vehicle’s final velocity increment \( \Delta u \) can be increased by a higher \( I_s \). This can be done by using a more energetic chemical propellant (see Chapters 7 and 12), by a higher chamber pressure, and, for upper stages operating at high altitudes, also by a larger nozzle area ratio. Electrical propulsion (higher \( I_s \)) can enhance vehicle performance, but, as explained later, the very low thrusts do limit this type to certain space missions. See Chapter 17.

2. The mass ratio \( m_0/m_f \) has a logarithmic effect. It can be increased in several ways. One way is by reducing the final mass \( m_f \), which consists of the inert hardware plus the nonusable, residual propellant mass. Reducing the inert mass implies lighter structures, smaller payloads, lighter guidance/controls devices, or less unavailable residual propellant; this means going to stronger structural materials at higher stresses, more efficient power supplies, or smaller electronic packages. During design there is always great emphasis to reduce all hardware masses and the residual propellants to their practical minima. Another way is to increase the initial vehicle mass, and use a higher thrust and more propellant, but with a smaller increase in the structure or inert propulsion system masses.

3. Reducing the burning time (i.e., increasing the thrust level) will reduce the gravitational loss in some applications. However, the higher acceleration usually requires more structural and propulsion system mass, which in turn causes the mass ratio to be less favorable.

4. The drag, which can be considered as a negative thrust, can be reduced in at least four ways. The drag has several components: (a) The form drag depends on the aerodynamic shape. A slender pointed nose or sharp, thin leading edges of fins or wings have less drag than a stubby, blunt shape. (b) A vehicle with a small cross-sectional area has less drag. A propulsion design that can be packaged in a long, thin shape will be preferred. (c) The drag is proportional to the cross-sectional or frontal vehicle area. A higher propellant density will decrease the propellant volume and therefore will allow a smaller cross section. (d) The skin drag is caused by the friction of the air flowing over all the vehicle’s outer surfaces. A smooth contour and a polished surface are usually better. The skin drag is also influenced by the propellant density, because it gives a smaller volume and thus a lower surface area. (e) The base drag is the fourth component; it is a function of the local ambient air pressure acting over the surface of the vehicle’s base or bottom plate. It is influenced by the nozzle exit design (exit pressure), the discharge of turbine exhaust gases, and the geometry of the vehicle base design. It is discussed further in Chapter 20.

5. The length of the propulsion nozzle often is a significant part of the overall vehicle or stage length. As was described in Chapter 3, there is an optimum nozzle contour and length, which can be determined by trade-off analysis. A shorter nozzle length or multiple nozzles on the same propulsion system allow a somewhat shorter vehicle; on many designs this implies a somewhat lighter vehicle structure and a slightly better vehicle mass ratio.

6. The final vehicle velocity at propulsion termination can be increased by increasing the initial velocity \( u_0 \). By launching a satellite in an eastward direction the rotational speed of the earth is added to the final satellite orbital velocity. This tangential velocity of the earth is about 464 m/sec or 1523 ft/sec at the equator and the Sea Launch from a ship on the equator takes full advantage of this velocity increment. For an easterly launch at John F. Kennedy Space Center (latitude of 28.5° north) this extra velocity is about 408 m/sec or 1340 ft/sec. Conversely, a westerly satellite launch has a negative initial velocity and thus requires a higher-velocity increment. Another way to increase \( u \) is to launch a spacecraft from a satellite or an aircraft, which increases the initial vehicle velocity and allows launching in the desired direction, or to launch an air-to-surface missile from an airplane. An example is the Pegasus three-stage space launch vehicle, which is launched from an airplane.

7. For vehicles that fly in the atmosphere it is possible to increase the range when aerodynamic lift is used to counteract gravity and reduce gravity losses. Using a set of wings or flying at an angle of attack increases the lift, but it also increases the drag. This lift can also be used to increase the maneuverability and trajectory flexibility.

8. When the flight velocity \( u \) is close to the rocket’s effective exhaust velocity \( c \), the propulsive efficiency is the highest (Eq. 2–23) and more of the rocket exhaust gas energy is transformed into the vehicle’s flight energy. Trajectories where \( u \) is close in value to \( c \) for a major portion of the flight therefore need less propellant.

Several of these influencing parameters can be optimized. Therefore, for every mission or flight application there is an optimum propulsion system design and the propulsion parameters that define the optimum condition are dependent on vehicle or flight parameters.

### 4.7. FLIGHT VEHICLES

As mentioned, the vast majority of rocket-propelled vehicles are single, simple stage, and use solid propellant rocket motors. Most are used in military applications, as described in the next section. This section discusses more sophisticated multistage space launch vehicles and mentions others, such as large ballistic missiles (often called strategic missiles) and some sounding rockets. All have some...
intelligence in their guidance and navigation system. The total number of multi-
stage rocket vehicles produced worldwide in the last few years has been between
100 and 220 per year.

A single stage to orbit (LEO) is limited in the payload it can carry. Figure 4—2
shows that a high-performance single-stage vehicle with a propellant fraction of
0.95 and an average $I_e$ of 400 sec can achieve an ideal terminal velocity of about
12,000 m/sec without payload. If the analysis includes drag and gravity forces, a
somewhat higher value of $I_e$, maneuvers in the trajectory, and an attitude control
system, it is likely that the payload would be between 0.2 and 1.4% of the gross
takeoff mass, depending on the design. For a larger percentage of payload, and
for ambitious missions, we use vehicles with two or more stages as described here.

Multistage Vehicles

Multistage or multistage rocket vehicles permit higher vehicle velocities, more
payload for space vehicles, and improved performance for long-range ballistic
missiles. After the useful propellant is fully consumed in a particular stage,
the remaining empty mass of that expended stage is dropped from the vehicle
and the operation of the propulsion system of the next step or stage is started.
The last or top stage, which is usually the smallest, carries the payload. The
empty mass of the expended stage or step is separated from the remainder of
the vehicle, because it avoids the expenditure of additional energy for further
accelerating a useless mass. As the number of steps is increased, the initial
takeoff mass can be decreased; but the gain in a smaller initial mass becomes less
apparent when the total number of steps is large. Actually, the number of steps
chosen should not be too large, because the physical mechanisms become more
numerous, complex, and heavy. The most economical number of steps is usually
between two and six, depending on the mission. Several different multistage
launch vehicle configurations have been used successfully and four are shown in
Fig. 4—14. Most are launched vertically, but a few have been launched from an
airplane, such as the three-stage Pegasus space vehicle. See Example 4—3.

The payload of a multistage rocket is essentially proportional to the takeoff
mass, even though the payload is only a very small portion of the initial mass. If
a payload of 50 kg requires a 6000-kg multistage rocket, a 500-kg payload would
require a 60,000-kg rocket unit with an identical number of stages and a similar
configuration with the same payload fraction and the same propellants. When the
operation of the upper stage is started, immediately after thrust termination of
the lower stage, then the total ideal velocity of a multistage vehicle of tandem
or series-stage arrangement is simply the sum of the individual stage velocity
increments. For $n$ stages in series (one on top of each other) the final velocity
increment $\Delta u_f$ is

$$\Delta u_f = \sum_{1}^{n} \Delta u = \Delta u_1 + \Delta u_2 + \Delta u_3 + \cdots$$  \hspace{1cm} (4—35)

The individual velocity increments are given by Eq. 4—6. For the simplified case
of a vacuum flight in a gravity-free field this can be expressed as

$$\Delta u_f = c_1 \ln(1/\text{MR}_1) + c_2 \ln(1/\text{MR}_2) + c_3 \ln(1/\text{MR}_3) + \cdots$$  \hspace{1cm} (4—36)

This equation defines the maximum velocity an ideal tandem multistage ve-
cle can attain in a gravity-free vacuum environment. For more accurate actual
trajectories the individual velocity increments can be determined by integrating
Eqs. 4—15 and 4—16, which consider drag and gravity losses. Other losses or
trajectory perturbations can also be included, as mentioned earlier in this chapter.
Such an approach requires numerical solutions.

For two- or three-stage vehicles the overall vehicle mass ratio (initial mass at
takeoff to final mass of last stage) can reach values of over 100
to an equivalent single-stage propellant mass fraction \( \xi \) of 0.99). Figure 4–2 can be thus divided into regions for single- and tandem multistage vehicles. Equation 4–36 does not apply to parallel or partial staging as identified in Fig. 4–14. For stages where more than one propulsion system is operating at the same time and producing thrust in the same direction, the effective specific impulse and nozzle exhaust velocity is given by Eqs. 10–12 to 10–14.

The first sketch in Fig. 4–14 depicts a very common configuration and the stages are stacked vertically on top of each other, as in the Minuteman long-range missile or the Russian Zenit (Zenith) launch vehicle.* Partial staging was used on the early versions of the U.S. Atlas vehicle. It allows all engines to be started at launching, thus avoiding an altitude start of the sustainer engine, which was unknown in those early days. Liquid propellant rocket engines can be shut off on the launch stand if a failure is sensed prior to lift-off. The two Atlas booster engines arranged in a doughnut-shaped assembly are dropped off in flight. The third sketch has two or more separate booster "strap-on" stages attached to the bottom stage of a vertical configuration (they can be either solid or liquid propellants) and this allows an increase in vehicle performance. The piggy-back configuration concept on the right is used on the Space Shuttle. The two large solid rocket motor boosters are not shown.

**Stage Separation**

It usually takes a finite time for the termination of the lower stage propulsion system to go from full thrust to essentially zero thrust (typically 1 to 3 sec for large thrust values). In some multistage flight vehicles (with stage separation devices) there often is a further short delay (say 4 to 10 sec) to achieve a respectable separation distance between the upper and the lower stage, before the firing of the upper stage propulsion system was initiated. This was done in order to prevent blow-back of damaging hot flames onto the upper stage. Also the upper stage engine start-up was not instantaneous but required one or more seconds. During this cumulative delay of several seconds the earth's gravity pull continues to diminish the vehicle's upward velocity, causing a reduction of the flight velocity by perhaps 20 to 500 ft/sec (7 to 160 m/sec). A scheme called **hot staging** has been used to diminish this velocity loss and shorten the staging time interval. The upper stage propulsion system is actually started at low but increasing thrust before the lower stage propulsion system has been fully shut off or well before it reaches essentially zero thrust. There are special flame-resistant ducts in the interstage structure to allow the flame or hot exhaust gases of the upper stage engine to be safely discharged and deflected symmetrically prior to and immediately after the actual separation of the stages without harming the functional hardware of the vehicle. This hot staging scheme has been used on large multistage vehicles, such as the Titan II in the United States and certain launch vehicles in China and the Soviet Union, because it does improve the flight performance.

For multistage vehicles the stage mass ratios, thrust levels, propulsion durations, and location or travel of the center of gravity of the stages are usually optimized, often using a complex trajectory computer program. The high specific impulse rocket engine (e.g., using hydrogen–oxygen propellants) is normally employed in upper stages of space launch vehicles, because a small increase in specific impulse can be more effective there than in lower stages.

---

*The three-stage Zenit space launch vehicle is assembled in the Ukraine with Russian rocket engines. It is transported to the United States, loaded on a special floating launch platform (Sea Launch Program), and launched by a team that is headed by the Boeing Company.*

Example 4–3. A two-stage exploration vehicle is launched from a high-orbit satellite into a gravity-free vacuum trajectory. The following notation is used and explained in the accompanying diagram as well as in Fig. 4-1:

- \( m_0 \) = initial mass of vehicle (or stage) at launch
- \( m_p \) = useful propellant mass of stage
- \( m_i \) = initial mass of stage(s)
- \( m_f \) = final mass after rocket operation; it includes the empty propulsion system with its residual propellant, the vehicle structures plus the propulsion system with control, guidance and payload masses
- \( m_{\text{pl}} \) = payload mass; it can include guidance, control and communications equipment, antennas, scientific instruments, military equipment, research apparatus, power supply, solar panels, sensors, etc.

---

[Diagram of a two-stage rocket vehicle showing mass fractions and stages.]
Subscripts 1 and 2 refer to the first and second stages. The following data are given:

- Flight velocity increment in gravity-free vacuum: 4700 m/sec
- Specific impulse (each stage): 310 sec
- Initial takeoff launch vehicle mass: 4500 kg
- Propellant mass fraction, \( \xi \) (each stage): 0.88

Determine the payload for the following cases:

1. When the two propulsion system or stage masses are equal \([m_1]_1 = [m_2]_1\).
2. When the mass ratios of the two stages are equal \([m_f]/[m_0] = [m_f]/[m_0]_2\).

**SOLUTION.** The following relationships apply to both cases. The takeoff mass or launch mass can be divided into three parts, namely, the two propulsion stages and then payload

\[
(m_0) = (m_{11}) + (m_{12}) + (m)_{pl}
\]

The propellants are 88% of their propulsion system mass, and this is representative of a number of stages.

\[
(m_p)_{11} = 0.88(m_1)_{11} \quad \text{and} \quad (m_p)_{12} = 0.88(m_1)_{12}
\]

The nozzle exit area ratio and the chamber pressure are the same in both stages and both cases. Thus the exhaust velocities are the same,

\[
c_1 = c_2 = c = \frac{I_{sp}}{g_0} = 310 \times 9.807 = 3040 \text{ m/sec}
\]

**Case 1.** The stage masses and sizes are the same, or

\[
(m_{11}) = (m_{12}) = m_i
\]

Equation 4–36 can be rewritten

\[
e^{\Delta u/c} = \left(\frac{1}{MR_1}\right) \left(\frac{1}{MR_2}\right) = \left(\frac{m_0 - (m_p)_{11}}{m_0}\right) \left(\frac{(m_{11})_2 + m_{pl} - (m_p)_{12}}{(m_{12})_2 + m_{pl}}\right)
\]

\[
e^{5500/3040} = \left(\frac{4500 - 0.88m_1}{4500}\right) \left(\frac{m_{12} + m_{pl} - 0.88m_1}{m_{12} + m_{pl}}\right) = 6.105
\]

From the equation above \( m_i = \frac{1}{2} \left(4500 - m_{pl}\right) \).

These two equations have only two unknowns: \( m_i \) and \( m_{pl} \). The solution requires a quadratic equation, and the value of the payload can be determined as \( m_{pl} \approx 480 \text{ kg} \).

**Case 2.** The mass ratios for the two stages are equal, or

\[
MR_1 = MR_2 = MR = \frac{m_0}{m_f}
\]

\[
1/MR = \left(\frac{4500 - (m_p)_{11}}{4500}\right) = \left(\frac{(m_{11})_2 + m_{pl} - (m_p)_{12}}{(m_{12})_2 + m_{pl}}\right)
\]

\[
e^{\Delta u/c} = \frac{4500 - 0.88(m_1)_1}{4500} = 0.12(m_{12})_2 + m_{pl} = e^{5500/3040} = 6.105
\]


These two equations (and the first equation of the example makes a third) have three variables: \( (m_1)_1 \), \( (m_1)_2 \), and \( m_{pl} \). The solution is possible and requires quadratic equations and natural logarithms. The result is a payload of approximately 600 kg.

If a three-stage vehicle had been used in Example 4-3 instead of a two-stage version, the payload would have been even larger. However, the theoretical payload increase will only be about 8 or 10%. A fourth stage gives an even smaller theoretical improvement; it would add only 3 to 5% to the payload. The amount of potential performance improvement diminishes with each added stage. Each additional stage means extra complications in an actual vehicle (such as a reliable separation mechanism, an interstage structure, more propulsion systems, joints or couplings in connecting pipes and cables, etc.), requires additional inert mass (increasing the mass ratio \( MR \)), and compromises the overall reliability. Therefore, the minimum number of stages that will meet the payload and the \( \Delta u \) requirements is usually selected.

The flight paths taken by the vehicles in the two simplified cases of Example 4-3 are different, since the time of flight and the acceleration histories are different. One conclusion from this example applies to all multistage rocket-propelled vehicles; for each mission there is an optimum number of stages, an optimum distribution of the mass between the stages, and usually also an optimum flight path for each design, where a key vehicle parameter such as payload, velocity increment, or range is a maximum.

**Launch Vehicles**

Usually the first or lowest stage, often called a booster stage, is the largest and it requires the largest thrust and largest total impulse. For earth surface launch all stages now use chemical propulsion to achieve the desired thrust-to-weight ratio. These thrusts usually become smaller with each subsequent stage, also known as upper stage or sustainer stage. The thrust magnitudes depend on the mass of the vehicle, which in turn depends on the mass of the payload and the mission. Typical actual configurations are shown by simple sketches in Fig. 4-14. There is an optimum size and thrust value for each stage in a multistage vehicle and the analysis to determine these optima can be quite complex.

Many launch vehicles with heavy payloads have one to six large strap-on stages, also called zero stages or half stages. They augment the thrust of the booster stage, which is started at about the same time. A schematic diagram is shown in the third sketch of Fig. 4-14. Solid propellant strap-on stages are common, such as the Atlas V shown in Fig. 1-13 or the Space Shuttle shown in Fig. 1-14. They are usually smaller in size than the equivalent liquid propellant strap-on (due to higher propellant density) and have less drag and usually a very toxic exhaust. Liquid propellant strap-on stages are used in the Delta IV heavy lift launch vehicle (see Fig. 1-12), in the first Soviet ICBM (intercontinental ballistic missile, 1950s), and several Soviet/Russian space launch vehicles. They
deliver higher specific impulse, which enhances vehicle performance, and require propellant filling at the launch site.

There is a variety of existing launch vehicles. The smaller ones are for low payloads and low orbits; the larger ones usually have more stages, are heavier and more expensive, and have larger payloads or higher mission velocities. The vehicle cost increases with the number of stages and the initial vehicle launch mass. Once a particular launch vehicle has been proven to be reliable, it is usually modified and uprated to allow improvements in its capability or mission flexibility. Each of the stages of a space launch vehicle can have several rocket engines, each for specific missions or maneuvers. The Space Shuttle system has 67 different rockets which are shown schematically in Fig. 1–14. In most cases each rocket engine is used for a specific maneuver, but in many cases the same engine is used for more than one specific purpose; the small reaction control thrusters in the Shuttle serve, for example, to give attitude control (pitch, yaw, and roll) during orbit insertion and reentry, for counteracting internal shifting of masses (astronaut movement, extendible arm), small trajectory corrections, minor flight path adjustments, docking, and precise pointing of scientific instruments.

The spacecraft is that part of a launch vehicle that carries the payload. It is the only part of the vehicle that goes into orbit or deep space and some are designed to return to earth. The final major space maneuver, such as orbit injection or planetary landing, often requires a substantial velocity increment; the propulsion system, which provides the force for this maneuver, may be integrated with the spacecraft or it may be part of aiscardable stage, just below the spacecraft. Several of the maneuvers described in Section 4.5 can often be accomplished by propulsion systems located in two different stages of a multistage vehicle. The selection of the most desirable propulsion systems, and the decision of which of the several propulsion systems will perform specific maneuvers, will depend on optimizing performance, cost, reliability, schedule, and mission flexibility as described in Chapter 19.

When a space vehicle is launched from the earth’s surface into an orbit, it flies through three distinct trajectory phases. (1) Most are usually launched vertically and then undergo a turning maneuver while under rocket power to point the flight velocity vector into the desired direction. (2) The vehicle then follows a free-flight (unpowered) ballistic trajectory (usually elliptical), up to its apex. Finally (3) a satellite needs an extra push from a chemical rocket system to add enough total impulse or energy to accelerate it to orbital velocity. This last maneuver is also known as orbit insertion or sometimes as a kick maneuver. During the initial powered flight the trajectory angle and the thrust cutoff velocity of the last stage are adjusted by the guidance system to a velocity vector in space that will allow the vehicle to reach the apogee of its elliptic path exactly at the desired orbit altitude. As shown in Fig. 4–8, a multistage ballistic missile follows the same two ascent flight phases mentioned above, but it then continues its elliptical ballistic trajectory all the way down to the target.

Historically successful launch vehicles have been modified, enlarged, and improved in performance. The newer versions retain most of the old, proven, reliable components, materials, and subsystems. This reduces development effort and cost. Upgrading a vehicle allows an increase in mission energy (more ambitious mission) or payload or both. Typically, it is done by one or more of these types of improvement: increasing the mass of propellant without an undue increase in tank or case mass; uprating the thrust and strengthening the engine; more specific impulse; or adding successively more or bigger strap-on boosters. It also usually includes a strengthening of the structure to accept higher loads.

Figure 4–15 shows the effects of orbit inclination and altitude on the payload. The inclination is the angle between the equatorial plane of the earth and the trajectory. An equatorial orbit has zero inclination and a polar orbit has 90° inclination. Since the earth’s rotation gives the vehicle an initial velocity, a launch from the equator in an eastward direction will give the highest payload. For the same orbit altitude other trajectory inclinations have a lower payload. For the same inclination the payload decreases with orbit altitude, since more energy has to be expended to overcome gravitational attraction.
Solid propellant rocket motors are preferred for most tactical missile missions, because they allow simple logistics and can be launched quickly. If altitudes are low and flight durations are long, such as with a cruise missile, an air-breathing jet engine and a winged vehicle, which provides lift, will usually be more effective than a long-duration rocket. However, a large solid propellant rocket motor may still be needed as a booster to launch the cruise missile and bring it up to speed.

For each of the tactical missile applications, there is an optimum rocket propulsion system and almost all of them use solid propellant rocket motors. Liquid propellant rocket engines have recently been used for the upper stages of two-stage anti-aircraft missiles and ballistic defense missiles, because they can be pulsed for different durations and can be randomly throttled. For each application there is a optimum total impulse, an optimum thrust time profile, an optimum nozzle configuration (single or multiple nozzles, with or without thrust vector control, optimum area ratio), optimum chamber pressure, and a favored solid propellant grain configuration. Low exhaust plume gas radiation emissions in the visible, infrared, or ultraviolet spectrum and certain safety features (making the system insensitive to energy stimuli) can be very important in some of the tactical missile applications; these are discussed in Chapters 13 and 20.

Short-range, uncontrolled, unguided, single-stage rocket vehicles, such as military rocket projectiles (ground and air launched) and rescue rockets, are usually quite simple in design. Their general equations of motion are derived in Section 4.3, and a detailed analysis is given in Ref. 4-1.

Unguided military rocket-propelled missiles are today produced in larger numbers than any other category of rocket-propelled vehicles. The 2.75-in.-diameter, folding fin unguided solid propellant rocket missile has recently been produced in the United States in quantities of almost 250,000 per year. Guided missiles for anti-aircraft, antitank, or infantry support have been produced in annual quantities of hundreds and sometimes over a thousand. Table 1-6 lists several guided missiles.

Because these rocket projectiles are essentially unguided missiles, the accuracy of hitting a target depends on the initial aiming and the dispersion induced by uneven drag, wind forces, oscillations, and misalignment of nozzles, body, and fins. Deviations from the intended trajectory are amplified if the projectile is moving at a low initial velocity, because the aerodynamic stability of a projectile with fins is small at low flight speeds. When projectiles are launched from an aircraft at a relatively high initial velocity, or when projectiles are given stability by spinning them on their axis, their accuracy of reaching a target is increased 2- to 10-fold, compared to a simple fin-stabilized rocket launched from rest.

In guided air-to-air and surface-to-air rocket-propelled missiles the time of flight to a given target, usually called the time to target $t_f$, is an important flight performance parameter. With the aid of Fig. 4-16 it can be derived in a simplified form by considering the distance traversed by the rocket (called the range) to be the integrated area underneath the velocity-time curve. This simplification assumes no drag, no gravity effect, horizontal flight, a relatively small distance traversed during powered flight compared to the total range, and a linear increase
FIGURE 4-16. Simplified trajectory for an unguided, nonmaneuvering, air-launched rocket projectile. Solid line shows ideal flight velocity without drag or gravity and dashed curve shows likely actual flight.

in velocity during powered flight:

\[ t_r = \frac{S + \frac{1}{2} u_p t_p}{u_0 + u_p} \]  \hspace{1cm} (4–37)

Here \( S \) is the flight vehicle’s range to the target and it is the integrated area under the velocity–time curve. Also \( u_p \) is the velocity increase of the rocket during powered flight up to the time of burnout, \( t_p \) is the time of rocket burning, and \( u_0 \) is the initial velocity of the launching aircraft. For the same flight time the range of the actual vehicle (dashed line) is less than for the ideal dragless vehicle. For more accurate values, the velocity increase \( u_p \) is given by Eq. 4–19. More accurate values can also be obtained through a detailed step-by-step trajectory analysis that considers the effects of drag and gravity.

In unguided air-to-air or air-to-surface rocket-powered projectiles the aiming at the target is done largely by orienting the launching aircraft into the direction of the target. A relatively simple solid propellant rocket motor is the most common choice for the propulsion. In guided missiles, such as air-to-air, air-to-ground, or ground-to-air, the flight path to the target has to be controlled and this can be achieved by controlling aerodynamic control surfaces and/or propulsion systems, which can be pulsed (repeated start and stop) and/or throttled to a lower thrust. The guidance system and the target seeker system of a guided missile will sense and track the flight path of a flying target, a computer will calculate a predicted impact point, and the missile’s flight control will change the flight path of the guided missile to achieve the impact with the intended target. The control system will command the propulsion system to operate or fire selected liquid propellant thrusters of an engine with multiple thrusters (or to selectively provide thrust through multiple nozzles with hot-gas shut-off valves in solid motors). A similar set of events can occur in a defensive ground-to-incoming-ballistic-missile scenario. It requires propulsion systems capable of pulsing or repeated starts, possibly with some throttling and side forces. Rocket engines with these capabilities are discussed in Section 6.8.

In both the unguided projectile and the guided missile the hit probability increases as the time to target \( t_r \) is reduced. In one particular air-to-air combat situation, the effectiveness of the rocket projectile varied approximately inversely as the cube of the time to target. The best results (e.g., best hit probability) are usually achieved when the time to target is as small as practically possible.

The analysis of the missile and propulsion configuration that gives the minimum time to target over all the likely flight scenarios can be complex. The following rocket propulsion features and parameters will help to reduce the time to target, but their effectiveness will depend on the specific mission, range, guidance and control system, and particular flight conditions.

1. High initial thrust or high initial acceleration for the missile to quickly reach a high-initial-powered flight velocity.

2. Application of additional lower thrust to counteract drag and gravity losses and thus maintain a high flight velocity. This can be a single rocket propulsion system that has a short high initial thrust and a smaller (10 to 25%) sustaining thrust of longer duration.

3. For higher supersonic flight speeds, a two-stage missile can be more effective. Here the first stage is dropped off after its propellant has been consumed, thus reducing the inertia mass of the next stage and increasing its mass ratio and thus its flight velocity increase.

4. If the target is highly maneuverable and if the closing velocity between missile and target is large, it may be necessary not only to provide an axial thrust but also to apply large side forces or side accelerations to a defensive tactical missile. This can be accomplished either by aerodynamic forces (lifting surfaces or flying at an angle of attack) or by multiple-nozzle propulsion systems with variable or pulsing thrusts; the rocket engine then would have an axial thruster and one or more side thrusters. The side thrusters have to be so located that all the thrust forces are essentially directed through the center of gravity of the vehicle in order to minimize turning moments. The thrusters that provide the side accelerations have also been called divert thrusters, since they divert the vehicle in a direction normal to the axis of the vehicle.

5. Drag losses can be reduced if the missile has a large \( L/D \) ratio (or a small cross-sectional area) and if the propellant density is high, allowing a smaller missile volume. The drag forces can be high if the missile travels at low altitude and high speed. A long and thin propulsion system geometry and a high-density propellant will help to reduce drag.

A unique military application is rocket-assisted gun-launched projectiles for attaining longer artillery ranges. Their small rocket motors withstand very high
accelerations in the gun barrel (5000 to 10,000 $g_0$ is typical). They have been in production.

### 4.9. FLIGHT STABILITY

Stability of a vehicle is achieved when the vehicle does not rotate or oscillate in flight. Unstable flights are undesirable, because pitch or yaw oscillations increase drag (flying at an angle of attack most of the time) and cause problems with instruments and sensors (target seekers, horizon scanners, sun sensors, or radar). Instability often leads to tumbling (uncontrolled turning) of vehicles, which causes missing of orbit insertion, missing targets, or sloshing of liquid propellant in tanks.

Stability can be built in by proper design so that the flying vehicle will be inherently stable, or stability can be obtained by appropriate controls, such as the aerodynamic control surfaces on an airplane, a reaction control system, or hinged multiple rocket nozzles.

Flight stability exists when the overturning moments (e.g., those due to a wind gust, thrust misalignment, or wing misalignment) are smaller than the stabilizing moments induced by thrust vector controls or by aerodynamic control surfaces. When the destabilizing moments exceed the stabilizing moments about the center of gravity, the vehicle turns or tumbles. In unguided vehicles, such as low-altitude rocket projectiles, stability of flight in a rectilinear motion is achieved by giving a large stability margin to the vehicle by using tail fins and by locating the center of gravity ahead of the center of aerodynamic pressure. In a vehicle with an active stability control system, a nearly neutral inherent stability is desired, so that the applied control forces are small, thus requiring small control devices, small RCS thrusters, small actuating mechanisms, and structural mass. Neutral stability is achieved by locating aerodynamic surfaces and the mass distribution of the components within the vehicle in such a manner that the center of gravity is just above the center of aerodynamic pressure. Because the aerodynamic moments change with Mach number, the center of pressure does not necessarily stay fixed during accelerating flight but shifts, usually along the vehicle axis. The center of gravity also changes its position as propellant is consumed and the vehicle mass decreases. Thus it is usually very difficult to achieve neutral missile stability at all altitudes, speeds, and flight conditions.

Stability considerations affect rocket propulsion system design in several ways. By careful nozzle design and careful installation it is possible to minimize thrust misalignment and thus to minimize undesirable torques on the vehicle and the reaction control propellant consumption. It is possible to exercise considerable control over the travel of the center of gravity by judicious design. In liquid propellant rockets, special design provisions, special tank shapes, and a careful selection of tank location in the vehicle afford this possibility. By using nozzles at the end of a blast tube, as shown in Fig. 15–6, it is possible to place the solid propellant mass close to the vehicle's center of gravity. Attitude control liquid engines with multiple thrusters have been used satisfactorily to obtain control moments for turning vehicles in several ways, as described in Section 4.5 and in Chapter 6.

Unguided rocket projectiles and missiles are often given a roll or rotation by inclined aerodynamic fins or inclined multiple rocket exhaust gas nozzles to improve flight stability and accuracy. This is similar to the rotation given to bullets by spiral-grooved barrels. This spin stability is achieved in part by gyroscopic effects, where an inclination of the spin axis is resisted by torques. The centrifugal effects cause problems in emptying liquid propellant tanks and extra stresses on solid propellant grains. In some applications a low-speed roll is applied not for spin stability but to assure that any effects of thrust vector deviations or aerodynamic vehicle shape misalignments are minimized and canceled out.

### PROBLEMS

1. For a vehicle in gravitationless space, determine the mass ratio necessary to boost the vehicle velocity by (a) 1600 m/sec and (b) 3400 m/sec; the effective exhaust velocity is 2000 m/sec. If the initial total vehicle mass is 4000 kg, what are the corresponding propellant masses?

**Answers:** (a) 2204 kg.

2. Determine the burnout velocity and burnout altitude for a dragless projectile with the following parameters for a simplified vertical trajectory: $v = 2200$ m/sec; $m_p/m_0 = 0.57$; $t_p = 5.0$ sec; and $u_0 = 0$; $h_0 = 0$. Select a relatively small diameter missile with $L/D$ of 10 and an average vehicle density of 1200 kg/m$^3$.

3. Assume that this projectile had a drag coefficient essentially similar to the 0° curve in Fig. 4–3 and redetermine the answers of Problem 3 and the approximate percentage errors in $u_0$ and $h_p$. Use a step-by-step or a numerical method.

4. A research space vehicle in gravity-free and drag-free outer space launches a smaller spacecraft into a meteor shower region. The 2-kg sensitive instrument package of this spacecraft (25 kg total mass) limits the maximum acceleration to no more than 50 m/sec$^2$. It is launched by a solid propellant rocket motor ($I_s = 260$ sec and $\xi = 0.88$). Assume instant start and stop of rocket motor.

   (a) Determine the maximum allowable burn time, assuming steady constant propellant mass flow.
   (b) Determine the maximum velocity relative to the launch vehicle.
   (c) Solve for (a) and (b) if half of the total impulse is delivered at the previous propellant mass flow rate, with the other half at 20% of this mass flow rate.

5. For a satellite cruising in a circular orbit at an altitude of 500 km, determine the period of revolution, the flight speed, and the energy expended to bring a unit mass into this orbit.

   **Answers:** 1.58 hr, 7613 m/sec, 33.5 MJ/kg.

6. A large ballistic rocket vehicle has the following characteristics: propellant mass flow rate: 12 slugs/sec (1 slug = 32.2 lbm = 14.6 kg); nozzle exit velocity: 7100 ft/sec;
nozzle exit pressure: 5 psia (assume no separation); atmospheric pressure: 14.7 psia (sea level); takeoff weight: 12.0 tons (1000 lbm); burning time: 50 sec; nozzle exit area: 400 in.². Determine (a) the sea-level thrust; (b) the sea-level effective exhaust velocity; (c) the initial thrust-to-weight ratio; (d) the initial acceleration; (e) the mass inverse ratio \( \frac{m_0}{m_f} \).

**Answers:** 81,320 lbf; 6775 ft/sec; 3.38; 2.38 g0.

7. In Problem 5 compute the altitude and missile velocity at the time of power plant cutoff, neglecting the drag of the atmosphere and assuming a simple vertical trajectory.

8. A spherical satellite has 12 identical monopropellant thrust chambers for attitude control with the following performance characteristics: thrust (each unit): 5 lbf; \( I_e \) (steady state or more than 2 sec): 240 sec; \( I_t \) (pulsing duration 20 m sec): 150 sec; \( I_t \) (pulsing duration 100 m sec): 200 sec; satellite weight: 3500 lbf; satellite diameter: 8 ft; satellite internal density distribution is essentially uniform; disturbing torques, Y and Z axes: 0.00005 ft-lbf average; disturbing torque, for X axis: 0.001 ft-lbf average; distance between thrust chamber axes: 8 ft; maximum allowable satellite pointing position error: ±1°. Time interval between pulses is 0.030 sec.

(a) What would be the minimum and maximum vehicle angular drift per hour if no correction torque were applied?

**Answers:** 0.466 and 0.093 rad/hr.

(b) What is the frequency of pulsing action (how often does an engine pair operate?) at 20-msec, 100-msec, and 2-sec pulses in order to correct for angular drift?

Discuss which pulsing mode is best and which is impractical.

9. For an ideal multistage launch vehicle with several stages in sequence, discuss the following: (a) the effect on the ideal mission velocity if the second and third stages are not started immediately but are each allowed to coast for a short period after shutoff and separation of the expended stage before rocket engine start of the next stage; (b) the effect on the mission velocity if an engine malfunctions and delivers a few percent less than the intended thrust but for a longer duration and essentially the full total impulse of that stage.

10. Given a cylindrical shaped space vehicle \( (D = 1 \text{ m}, \text{height is} 0.7 \text{ m}, \text{average density is} 1.1 \text{ g/cm}^3) \) with a flat solar cell panel on an arm (mass of 32 kg, effective moment arm is 1.5 m, effective average area facing normally toward sun is 0.6 m²) in a set of essentially frictionless bearings and in a low orbit at 160 km altitude with sunlight being received, on the average, about 60% of the period:

(a) Compute the maximum solar pressure-caused torque and the angular displacement this would cause during 1 day if not corrected.

(b) Using the data from the atmospheric table in Appendix 2 and an arbitrary average drag coefficient of 1.0 for both the body and the flat plate, compute the drag force and torque.

(c) Using stored high-pressure air at \( 14 \times 10^6 \text{ N/m}^2 \) initial pressure as the propellant for attitude control, design an attitude control system to periodically correct for these two disturbances \( (F, I_t, I, t, \text{etc.}) \).

(d) If the vector of the main thrust rocket of the vehicle (total impulse of \( 67 \times 10^3 \text{ N-sec} \)) is misaligned and misses the center of gravity by 2 mm, what correction would be required from the attitude control system? What would need to be done to the attitude control system in c above to correct for this error also?

11. Determine the payload for a single-stage vehicle in Example 4–3. Using the data from this example compare it with the two-stage vehicle.

12. An earth satellite is in an elliptical orbit with the perigee at 600 km altitude and an eccentricity of \( e = 0.866 \). Determine the parameters of the new satellite trajectory, if a rocket propulsion system is fired in the direction of flight giving an incremental velocity of 200 m/sec when (a) fired at apogee, (b) fired at perigee, and (c) fired at periapsis, but in the opposite direction, reducing the velocity.

13. A sounding rocket (75 kg mass, 0.25 m diameter) is speeding vertically upward at an altitude of 5000 m and a velocity of 700 m/sec. What is the deceleration in multiples of \( g \) due to gravity and drag? (Use \( C_D \) from Fig. 4–3 and use Appendix 2.)

14. Derive Eq. 4–37; state all your assumptions.

**SYMBOLS**

- \( a \) major axis of ellipse, m, or acceleration, m/sec² (ft/sec²)
- \( A \) area, m² (ft²)
- \( b \) minor axis of ellipse, m
- \( B \) numerical value of drag integral
- \( c \) effective exhaust velocity, m/sec (ft/sec)
- \( \varepsilon \) average effective exhaust velocity, m/sec
- \( C_D \) drag coefficient
- \( C_L \) lift coefficient
- \( d \) total derivative
- \( D \) drag force, N (lb)
- \( e \) eccentricity of ellipse, \( e = \sqrt{1 - b^2/a^2} \)
- \( E \) base of natural logarithm (2.71828)
- \( F \) energy, J
- \( F_f \) thrust force, N (lb)
- \( F_g \) final thrust, N
- \( F_0 \) gravitational attraction force, N
- \( F_N \) initial thrust force, N
- \( g \) gravitational acceleration, m/sec²
- \( g_0 \) gravitational acceleration at sea level, 9.8066 m/sec²
- \( G \) universal or Newton's gravity constant, 6.6700 x 10⁻¹¹ m³/kg-sec²
- \( h \) altitude, m (ft)
- \( h_p \) altitude of rocket at power cutoff, m
- \( I_s \) specific impulse, sec
- \( k_d \) diffuse coefficient of reflectivity
- \( k_s \) specular coefficient of reflectivity
- \( l \) distance of moment arm, m
**Greek Letters**

- \( \alpha \): angle of attack, deg or rad, or angular acceleration, angle/sec^2
- \( \xi \): propellant mass fraction (\( \xi = m_p/m_0 \))
- \( \theta \): angle between flight direction and horizontal, or angle of incidence of radiation, deg or rad
- \( \mu \): gravity constant for earth, \( 3.98600 \times 10^{14} \) m^3/sec^2
- \( \rho \): mass density, kg/m^3
- \( \tau \): period of revolution of satellite, sec
- \( \psi \): angle of thrust direction with horizontal
- \( \omega \): angular speed, deg/sec (rad/sec)

**Subscripts**

- \( e \): escape condition
- \( f \): final condition at rocket thrust termination
- \( i \): initial condition
- \( max \): maximum
- \( p \): condition at power cutoff or propulsion termination
- \( pl \): payload
- \( s \): satellite
- \( z \): zenith
- \( 0 \): initial condition or takeoff condition

**REFERENCES**


