Question 1: [Rocket]et Propulsion (10%):
An astronaut is accidentally separated from his spaceship. Luckily, during his mission he collected a bag of moon rocks. In order to make it back to the ship the astronaut begins to throw the rocks and moves back to safety.

At a given instant in time the following parameters are known:
- $R = \text{throwing rate (rocks/second)}$
- $Mastro = \text{mass of the astronaut and everything he carries}$
- $Mrock = \text{mass of a single rock and all rockets are identical}$
- $Vrock = \text{velocity of the rock relative to the astronaut}$
- $Vastro = \text{velocity of the astronaut and everything attached to him}$

**Part 1:** Using these variables, write down an expression for the force on the astronaut.

**Part 2:** In physical terms, how is the astronaut generating thrust?

\[
F = m U_{rock}\]

\[
F = R \left( \frac{\text{rocks}}{\text{sec}} \right) Mrock \left( \text{kg} \right) U_{rock} \left( \frac{\text{m}}{\text{s}} \right)
\]

\[
F = 2 Mrock U_{rock}
\]

**Astronaut is "thrusting" by reaction force.**
Question 2 (10 Points):
A person on a dock throws rocks to a person in a boat who in turn throws them into the water, as shown in the sketch below:

The following values are known:
- $R$=throwing rate (rocks/second, the same for both individuals)
- $M_{\text{boat}}$=Mass of the boat and everything in it
- $M_{\text{rock}}$=Mass of one rock
- $U_{\text{in}}$=Velocity of rock coming into the boat, relative to the boat
- $U_{\text{out}}$=Velocity of rock exit the boat, relative to the boat
- $U_{\text{boat}}$=Velocity of the boat

**Part 1:** Using any of the variables above, what is the **magnitude** and **direction** of the force on the boat?
**Part 2:** In physical terms, how is the boat being propelled through the water?

\[
F = m \Delta \text{U}
\]
\[
F = m(U_{\text{out}} - U_{\text{in}})
\]
\[
m = R M_{\text{rock}}
\]

\[
F = R M_{\text{rock}}(U_{\text{out}} - U_{\text{in}}) \text{ Magnitude}
\]

**Direction cannot be determined without actual values for $U_{\text{out}}$ and $U_{\text{in}}$**

**Propulsive force is produced by reaction to change in momentum flux of rocks being thrown into vs. out of boat (Control Volume)**
Question 3 (10 Points):
The effect of boundary layers within the throat of a rocket nozzle is an issue of active research. The throat of a nozzle is precisely machined such that its diameter is 12.00 cm. The rocket is placed on a thrust stand and the characteristic velocity is measured to be 1,000 m/s, with a chamber pressure of 1.00 MPa and a measured mass flow of 10.00 kg/s. About how much of the flow area is being ‘lost’ due to the presence of the boundary layer?

\[ D^* = 12 \text{ cm} \]
\[ A^* = 0.0113 \text{ m}^2, \text{ as measured} \]
\[ C^* = 1000 \text{ m/s} \]

\[ C^* = \frac{P^* A^*}{m} = \frac{(10^6)(A^*)}{10} \]
\[ A^* = 0.01 \text{ m}^2 \]

So,
\[ \frac{0.0113 - 0.01}{0.01} = 13\% \text{ loss in flow area} \]
**Question 4 (10 Points):**
Consider two different engines, **Viking 5C** and **Viking 4B**, used on the Ariane 4 rocket.

The motors use storable propellants called nitrogen tetroxide and UDMH25 (unsymmetrical dimethyl hydrazine with 25% hydrazine hydrate). The mixture is self-igniting and the constituents are liquid at standard temperature and pressure. The following information is available for the Viking 5C and Viking 4B engines:

<table>
<thead>
<tr>
<th>Engine</th>
<th>Viking 5C</th>
<th>Viking 4B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum thrust</td>
<td>752 kN</td>
<td>805 kN</td>
</tr>
<tr>
<td>Sea-level thrust</td>
<td>678 kN</td>
<td>N/A</td>
</tr>
<tr>
<td>Specific impulse</td>
<td>278.4 seconds</td>
<td>295.5 seconds</td>
</tr>
<tr>
<td>Chamber pressure</td>
<td>58 bar</td>
<td>58.5 bar</td>
</tr>
<tr>
<td>Area ratio</td>
<td>10.5</td>
<td>30.8</td>
</tr>
<tr>
<td>Mass flow</td>
<td>275.2 kg/sec</td>
<td>278.0 kg/sec</td>
</tr>
<tr>
<td>Nozzle exit diameter</td>
<td>0.990 meters</td>
<td>1.700 meters</td>
</tr>
</tbody>
</table>

Calculate the thrust coefficients for these two engines and briefly discuss their difference.

\[ (C_T)_{VAC} = \frac{\tau}{PA^*} \]

\[ A_c = \frac{\Pi D^2}{4} = 0.77 \text{ m}^2 \]

\[ (C_T)_{VAC} = \frac{752 \times 10^3}{(58 \times 10^5)(0.77)} = 1.769 \]

\[ (C_T)_{VAC} = \frac{805 \times 10^3}{(58.5 \times 10^5)(2.27)} = 1.867 \]

**4B is slightly better because of higher mean pressure and expansion ratio.**
**Question 5 (20 Points):**
A rocket has the following parameters:
- Thrust = 6,000 N
- Isp = 200 seconds
- Initial mass = 200 kg
- Burn time = 10 seconds

Assume that the exit pressure is equilibrated with the atmospheric back pressure at all times, neglect drag and assume gravity is a constant (use $g = 10 \text{ m/s}^2$ if you want).

1. The condition of maximum propulsive efficiency for an aircraft jet engine is the case where the flight speed is equal to the exhaust speed of the material exiting the engine. Does this rocket see such a condition of maximum propulsive efficiency at any time during a vertical launch?
2. What is the velocity at burnout?
3. How much additional height does the rocket attain after burnout?
4. **Extra Credit** (do this part only if you have completed the rest of the exam). If the burn time may be taken as much longer than 10 seconds and the propellant mass is kept constant, at what time in the flight does the condition of maximum propulsive efficiency occur?
\[ T = 6000 \text{ N} \]
\[ I_{sp} = 200 \text{ s} \]
\[ m_0 = 200 \text{ Kg} \]
\[ t_f = 10 \text{s} \]

**NEGLIGE DRAG**
- \( \rho = \rho_0 \)
- \( g = 10 \text{ m/s}^2 \)

**DOES \( V \) EVEN EQUAL \( V_e \)?**

\[ U = U_e \ln \left( \frac{m_0}{m_0 - m_0} \right) - gt \]

\[ I_{sp} = \frac{U_e}{g} \quad ; \quad U_e = I_{sp}g = (200s)(10 \text{ m/s}^2) = 2000 \text{ m/s} \]

\[ T = \dot{m}V_e \quad ; \quad \dot{m} = \frac{T}{U_e} = \frac{6000 \text{ N}}{2000 \text{ m/s}^2} = 3 \text{ Kg/s} \]

IF \( V = V_e \), \( \frac{V}{V_e} = 1 \)

\[ I = \ln \left( \frac{m_0}{m_0 - \dot{m}t} \right) - \frac{gt}{V_e} \]

\[ I_1 = \ln \left( \frac{200 \text{ Kg}}{200 - (3 \times 10)} \right) - \frac{(10 \times 10)}{2000} \]

\[ I_1 = 0.1625 - 0.05 = 0.1125 \quad \text{NO, NEVER SEES} \quad U = U_e \]

**AT BURNOUT** \( U_b = 2000 \ln \left( \frac{200}{200 - 3t} \right) - (10 \times 10) = 225 \text{ m/s} \), \( \text{NO MOST} = \frac{U_b^2}{2g} = 2.5 \)

**NOTE, IF BURN TIME WAS LONGER WHEN WOULD THIS OCCUR**

\[ I = \ln \left( \frac{200}{200 - 3t} \right) - \frac{10t}{2000} \]

\[ t = 20s \quad I_2 = 0.257 \quad \text{NO, TOO SHORT} \]

\[ t = 50s \quad I_2 = 1.136 \quad \text{TOO LONG} \]

\[ t = 45s \quad I_2 = 0.9 \quad \text{ALMOST} \]

\[ t = 47s \quad I_2 = 0.99 \]

\[ t = 47.5s \quad I_2 = 1.009 \quad \text{THE EFFICIENCY CONDITION WOULD OCCUR ~ 47.5s INTO VERTICAL FLIGHT (NO DRAG)} \]
Question 6 (20 Points):
A portion of a nuclear thermal rocket engine for a Mars mission is shown below. The ‘nozzle skirt extension’ is attached to the initial diverging portion of the nozzle by a flange. The mass of the nozzle skirt extension is 500 kg and the engine is providing a thrust to accelerate the rocket at 5 m/s². The mass flow at the choked location of the nozzle is 100 kg/s. Is the connecting flange in tension or in compression, and that is the magnitude of that compressive or tensile force?

**At flange**
- Cross sectional area = 0.02 m²
- Pressure = 8 MPa
- Velocity = 400 m/s

**At exit plane of nozzle skirt extension**
- Cross sectional area = 1 m²
- Pressure = 0.1 Mpa
- Velocity = 2,500 m/s

\[
F = F_{\text{Flange (Assume tensile)}}
\]

\[
\sum F_y = 550\mu_y(U_a \cdot a)
\]

\[
F_{\text{Flange}} - mg - PaA_a + P_bA_b = -U_a(-U_a)\rho A_a - U_b(U_b)\rho A_b
\]

\[
F_{\text{Flange}} = \rho U_a - \rho U_b + PaA_a - P_bA_b + mg
\]

\[
= 100(400 - 250) + (8\times10^5)(0.02) - (1\times10^5)(1) + (500)(5)
\]

\[
F_{\text{Flange}} = -147.5 \text{ kN}
\]

**Where (+) is in tension**

So flange is in compression!
Question 7 (20 Points):
A new solid propellant is tested in a configuration which yields a chamber pressure of 1,000 psi with a nozzle that produces an exit pressure that matches the 1 atmosphere backpressure. The measured specific impulse is 265 seconds. The specific heat ratio is estimated to be 1.2. In a second experiment, the supersonic portion of the nozzle is replaced by one with an expansion ratio area ratio of $A_d/A_e=100$, and the test is repeated in vacuum.

1. How does $c^*$ vary between these two experiments?
2. What is the specific impulse in the second experiment?

Note: The pressure ratio corresponding to $A_d/A_e=100$ and $\gamma=1.2$ is $P_d/P_e=1/1521$.

\[
C^* = \frac{P_o A^*}{m}
\]

MUST REMAIN THE SAME IN BOTH CASES, WE ARE ONLY CHANGING DOWNSTREAM PROPERTIES.

\[
I_{sp} = \frac{T}{g m} = \frac{c^* c T}{g m^*} = \frac{c^* c T}{g m} = \frac{P_o A^*}{c T}
\]

SO THIS AMOUNTS TO FINDING C T'S.

\[
C_t = \sqrt{\frac{2 \gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1}\right)^{\left(\gamma + 1\right)/(\gamma - 1)} \left[1 - \left(\frac{P_e}{P_o}\right)^{(\gamma + 1)/\gamma}\right] + \frac{P_e - P_o}{P_e} \frac{A_e}{A^*}}
\]

CASE 1: $P_e = P_o = 1$ ATM

\[
(C_t)_1 = \sqrt{5.047 \left[1 - \left(\frac{14.7}{1000}\right)^{2/1.2}\right]} = 1.58
\]

CASE 2: $P_o = 0$, $P_e = 0.657$ atm

\[
(C_t)_2 = \sqrt{5.047 \left[1 - \left(\frac{0.657}{1000}\right)^{2/1.2}\right] + \frac{0.657 - 0}{1000} (100)}
\]

\[
= 1.9521
\]

\[
\frac{(C_t)_1}{(C_t)_2} = \frac{1.58}{1.9521} = 0.8094
\]

$\therefore (I_{sp})_2 = 327$ sec