Exam is worth 100 points and has 4 questions (30/30/25/25). Read each question carefully and show all your work.

**Question 1 (30 Points):**
An F-4 Phantom is being refueled in mid-air by a KC-10 Tanker. The refueling boom enters at an angle of 30° from the F-4 flight path. The fuel flow rate through the boom is 20 kg/s at a velocity of 30 m/s relative to the two aircraft. The density of the jet fuel is less than water, and is about 700 kg/m³. What additional lift force is necessary to overcome the force on the F-4 fighter due to the momentum transfer during refueling?

\[ \dot{m} = 20 \text{ kg/s} \]
\[ \rho = 700 \text{ kg/m}^3 \]

\[ \rho \left( -30 \sin 30° \right) \cdot 30 \cos 30° = F_y \]
\[ \rho \left( 30 \right)^2 \sin 30° \cos 30° = F_y \]
\[ = \rho V^2 \sin 30° \cos 30° = F_y \]

\[ F_y = \dot{m} V \cos 30° = 500 \text{ N} \]
Question 2 (30 Points):
We wish to operate a low-speed (incompressible, inviscid) wind tunnel so that the flow in
the test section has a velocity of 200 MPH (90 m/s). Consider two different types of wind
tunnels, (a) and (b):

a) A nozzle and a constant-area test section, where the flow at the exit of the test
section simply dumps out to the surrounding atmosphere (no diffuser).

b) An arrangement of nozzle, test section, and diffuser where the flow at the exit
of the diffuser dumps out to the surrounding atmosphere.

For both of these wind tunnels calculate the pressure differences across the entire wind
tunnel required to operate them so that they have velocity of 90 m/s in the test section.
The geometry of the nozzle inlet and test section is the same in both tunnels (a) and (b)
and is 2.0 m² and 0.4 m², respectively. For tunnel (b) only, we add a diffuser down stream
of the test section, and the diffuser has an exit area of 1.8 m². What does this say about
the value of adding a diffuser on a subsonic wind tunnel?

\[ P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2 \quad A_1 V_1 = A_2 V_2 \quad V_L = V_0 \frac{A_2}{A_1} \]

\[ P_1 + \frac{1}{2} \rho V_1^2 (\frac{A_2}{A_1})^2 = P_2 + \frac{1}{2} \rho V_2^2 \]

\[ P_1 - P_2 = \frac{1}{2} \rho V_L^2 (1 - (\frac{A_2}{A_1})^2) = \frac{1}{2} (1.2)(90)^2 \left(1 - \left(\frac{9}{2}ight)^2\right) \]

\[ \Delta P = P_1 - P_2 = 4665.6 \text{ Pa} \]

\[ P_1 + \frac{1}{2} \rho V_1^2 = P_3 + \frac{1}{2} \rho V_3^2 \quad A_1 V_1 = A_2 V_2 = A_3 V_3 \quad V_3 = V_0 \left(\frac{A_2}{A_3}\right) \]

\[ P_1 - P_3 = \frac{1}{2} \rho V_L^2 (\frac{A_2}{A_3})^2 - \frac{1}{2} \rho V_3^2 = \frac{1}{2} \rho V_3^2 \left(\frac{A_2}{A_3}\right)^2 - \frac{1}{2} \rho V_3^2 \left(\frac{A_2}{A_3}\right)^2 \]

\[ P_1 - P_3 = \frac{1}{2} \rho V_3^2 \left(\frac{A_2}{A_3} - \frac{A_2}{A_3}\right) = \frac{1}{2} (1.2)(90)^2 \left((\frac{9}{1.8})^2 - (\frac{9}{2})^2\right) \]

\[ \Delta P = P_1 - P_3 = 45.6 \text{ Pa} \]

\text{**Note:** Less \( \Delta P \) w/ diffuser.}

\text{Simulation 30, Fall 95, 8."
Question 3 (25 Points):
A steady, inviscid, incompressible flow has a velocity field given by:

\[ u = fx \quad v = -fy \quad w = 0 \]

Where \( f \) is a constant having the dimensions of \( 1/s \). Derive an expression for the pressure field \( p(x,y,z) \) if the pressure \( p(0,0,0) = p_0 \) and \( \mathbf{g} = -g \mathbf{k} \).

Velocity and Pressure Relation Through Momentum Equations:

X-direction:

\[ m \frac{du}{dx} + v \frac{du}{dy} = -\frac{d}{dx} \frac{d}{dx} \]

\[ f_x (x) + (-f_y)(y) = -\frac{1}{\rho} \frac{d}{dx} \]

\[ \frac{f_x}{f_x} = -\frac{1}{\rho} \frac{d}{dx} \quad (1) \]

Y-direction:

\[ m \frac{dv}{dx} + v \frac{dv}{dy} = -\frac{d}{dy} \frac{d}{dy} \]

\[ f_x (0) + (-f_y)(-y) = -\frac{1}{\rho} \frac{d}{dy} \]

\[ \frac{f_y}{f_y} = -\frac{1}{\rho} \frac{d}{dy} \quad (2) \]

Z-direction:

\[ 0 = -\frac{1}{\rho} \frac{d}{dz} - g \quad (3) \]

Integrate (1):

\[ \frac{p}{\rho} = -\frac{f_x}{2} + f(y,z) \]

Differentiate w.r.t. \( y \):

\[ \frac{1}{\rho} \frac{dp}{d\mathbf{y}} = 0 \quad \Rightarrow \quad f'(y,z) = -f_y \]

\[ \Rightarrow \quad f = -\frac{f_y}{2} + f(z) \]

So far:

\[ \frac{p}{\rho} = -\frac{f_x}{2} - \frac{f_y}{2} + f(z) \]

Differentiate w.r.t. \( z \):

\[ \frac{1}{\rho} \frac{dp}{dz} = f'(z) \quad \Rightarrow \quad f'(z) = -g \quad \Rightarrow \quad f(z) = -gz + C \]

\[ \frac{p}{\rho} = -\frac{f_x}{2} - \frac{f_y}{2} - gz + C \]

\[ p = p_0 - pgz - p \frac{f_x^2}{2} (x^2 + y^2) \]
Question 4 (25 Points):
Consider the two-dimensional, incompressible velocity potential given below:

\[ \phi = xy + x^2 - y^2 \]

a) Is this potential function a solution to Laplace’s Equation? If so, what does it mean?

b) If it exists, find the stream function \( \psi(x,y) \) for this flow.

c) Find the equation of the streamline that passes through \((x,y) = (2,1)\).

a) \[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2 + (-2) = 0 \]

This means flow is also incompressible and a stream function exists.

b) \[ \begin{align*}
\psi &= \frac{y^2}{2} + 2xy + C \quad \text{(for } \frac{\partial \psi}{\partial y} \text{)} \\
\frac{\partial \psi}{\partial x} &= 0 \\
\frac{\partial \psi}{\partial y} &= x - 2y \\
\frac{\partial \psi}{\partial x} &= 2x + 6'y
\end{align*} \]

\[ x - 2y = -2y - 5'(y) \]

\[ 5'(y) = -x \rightarrow 5(y) = -\frac{x^2}{2} + C \]

\[ \psi = \frac{y^2}{2} + 2xy - \frac{x^2}{2} + C = \frac{1}{2}(y^2 - x^2) + 2xy + C = \psi \]

c) \( \psi \) at \((x,y) = (2,1)\):

\[ \psi = \frac{1}{2}(1 - 4) + 2(2)(1) = -\frac{3}{2} + \frac{4}{2} = \frac{1}{2} \]

\[ \psi = \frac{5}{2} = \text{constant} \]

\[ \psi = \frac{1}{2}(y^2 - x^2) + 2xy = \frac{5}{2} \]
Extra Credit (10 points):
Only do the extra credit if you have completed the rest of the exam

Consider two different flows over geometrically similar airfoil shapes, one airfoil being twice the size of the other. The flow over the smaller airfoil has freestream properties given by $T_s=200K$, $\rho_s=1.23$ kg/m$^3$, and $V_s=100$ m/s. The flow over the larger airfoil is described by $T_s=800K$, $\rho_s=1.74$ kg/m$^3$, and $V_s=200$ m/s. Assume that the viscosity is proportional to $T^{1/2}$. Are the two flows dynamically similar?

**Dynamic similarity requires identical:**

1) **Mach Numbers**
2) **Reynolds Numbers**

1) **Mach**

\[
\frac{M_1}{M_2} = \frac{V_1}{a_1} \cdot \frac{a_2}{V_2} = \frac{a_1}{a_2} = \frac{1}{\sqrt{T_1}} \cdot \frac{100}{\sqrt{800}} = \frac{1}{1.23} \cdot \frac{100}{800} = 1
\]

Mach No's are same

2) **Reynolds**

\[
\frac{D_1}{D_2} = \frac{\rho V s c t (V s)}{\rho_2 V_2 c_2} = \frac{\rho_1 V_1 s c t (V_1 s)}{\rho_2 V_2 c_2} \sqrt{\frac{T_2}{T_1}}
\]

\[
= \frac{(1.23)(100)(100)}{(1.74)(200)(200)} \sqrt{\frac{800}{200}}
\]

\[
= 0.354
\]

Dc's are not identical

Flows are not dynamically similar.