We have spent a good amount of time studying the following flow field in detail:

\[ u = a(x^2 - y^2) \quad v = -2axy \quad w = 0 \]

We have shown that the velocity field is steady, incompressible, and irrotational. If we now add the assumption that the flow is also inviscid, as well as neglecting any external or body forces, the velocity field is subject to Newton's second law through the momentum equation, given in vector form below:

\[ \frac{D\vec{V}}{Dt} = -\frac{1}{\rho} \nabla p \]

For this concept quiz, determine if the given velocity field given is a solution to the inviscid momentum equation.

Perhaps the quickest and easiest way to do this is:

1) Evaluate the momentum equation in the x- and y-directions

2) Compare the second-order mixed partial derivatives of the pressure: \( \frac{\partial^2 p}{\partial x \partial y} \). If this term is identical in both directions (x and y), then the velocity field is a solution to Equation 1. If the mixed derivatives are not identical, the flow field given above is not a solution of Equation 1. Also note that: \( \frac{\partial^2 p}{\partial x \partial y} = \frac{\partial^2 p}{\partial y \partial x} \).

**X-DIRECTION**

\[ \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} = (a x^2 - a y^2)(2ax) + (-2axy)(-2ay) \]

\[ 2a^2 x^3 - 2a^2 x y^2 + 4a^2 x y^2 = \frac{-1}{\rho} \frac{\partial p}{\partial x} = 2a^2 (x^3 + x y^2) \quad (1) \]

**Y-DIRECTION**

\[ \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} = (a x^2 - a y^2)(-2ay) + (-2axy)(-2ax) \]

**CHECK** \( \frac{\partial^2 p}{\partial x \partial y} = ? \frac{\partial^2 p}{\partial y \partial x} \)

\[ = -2a^2 x^2 y + 2a^2 y^3 + 4a^2 x y^2 = 2a^2 (x^2 y + y^3) \quad (2) \]

From 1: \( \frac{\partial^2 p}{\partial x \partial y} = 4a^2 xy \)

From 2: \( \frac{\partial^2 p}{\partial y \partial x} = 4a^2 xy \)

*It is a solution*