Modeling and Validation of Ku-Band Signal Attenuation Through Rocket Plumes

Bartel van der Veek,∗ Sunil Chintalapati,‡ Daniel R. Kirk,‡ and Hector Gutierrez‡

Florida Institute of Technology, Melbourne, Florida 32901
and
Rudy F. Bun‡
Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands

DOI: 10.2514/1.A32358

Communications to and from a launch vehicle during ascent are of critical importance to the success of rocket-launch operations. During ascent, the rocket’s exhaust plume causes significant interference in the radio communications between the vehicle and ground station. This paper presents an improved line of sight: a lower-collision-frequency microwave signal attenuation model based on the plume chemistry, microwave path, and the cross section for momentum transfer. The proposed model considers the combustion products in the exhaust plume, ambient air entrainment downstream of the nozzle, and a plasma attenuation model to estimate the total radio-frequency signal attenuation. The proposed model is benchmarked against measurements of signal attenuation through an exhaust plume in the 12.4 to 18 GHz frequency range.

I. Introduction

T HE safety and success of launch operations depends (among other factors) on the ability to maintain radio contact with a rocket during its launch. For example, in the event of a potentially catastrophic situation, the rocket must properly receive vehicle destruct kill signals. During the rocket’s ascent phase, the exhaust plume may cause significant disturbances in the radio communication between rocket and ground station and, in some cases, even lead to radio silence [1]. Existing models attempt to estimate signal attenuation due to the presence of the exhaust plume, but do not take advantage of computational tools that link plume chemistry to signal transmission. Furthermore, current models lack fidelity due to oversimplification of some critical physical parameters, such as the cross section for momentum transfer. To compensate for model uncertainty, transmission equipment onboard the rocket is often oversized in terms of power, with a corresponding mass penalty that translates to reduced payload capacity. An improved model of signal attenuation through the rocket’s plume could serve to more properly size the communications equipment onboard the vehicle, leading to cost and mass savings as well as improved reliability.

Several models have been developed to describe the interaction of a microwave signal traveling through a rocket exhaust plume. The composition of the plume is a function of pressure, temperature, and the initial mixture of reactants; that is, fuel and oxidizer. The plume is not uniform and contains several different flow regions with different gradients of density, temperature, pressure, velocity, and composition of product species. Usually, it is visible as a brilliant flame that emits radiation energy in the infrared, visible, and ultraviolet segments of the spectrum. Plume signal attenuation models can be categorized in three groups: diagonal refraction models, diagonal diffraction models, and line-of-sight (LOS) attenuation models. The models predict signal attenuation based on the collision characteristics of the exhaust plume, and most only apply over a limited range of frequencies and plume compositions. For example, Smoot and...
Underwood [2] state that current LOS models are inaccurate in predicting signal attenuation for rocket exhaust plumes with more than 5% aluminum content. Victor [3] compares several models and concludes that the diffraction model, which is based on measurements at a single frequency at several points and angles through the plume, describes the microwave attenuation best. Nayak and Thomas [4] present a theoretical analysis to estimate electrical parameters, such as electrical conductivity and permittivity, and their distribution in the axial and radial directions of the exhaust plume of an airborne vehicle. Williams [5] gives a full description of a computer model developed to describe amplitude and phase noise spectra generated by incoherent volume scattering in a turbulent ionized exhaust jet.

Seliga and Smoot [6] present experimental results from a set of measurements where X-band attenuation is measured simultaneously with amplitude/phase modulation (AM/PM) noise for seven different double-base propellant systems and a single rubber-base system. Frederick and Blevins [7] describe an experimental investigation using multiple-frequency microwave attenuation measurements in laboratory-scale solid-rocket exhaust plumes for ranking propellants according to their production of free electrons.

This paper describes an improved line-of-sight lower collision frequency (LOS-LCF) model of the plume signal attenuation. Much of the previous work on predicting attenuation through rocket plumes presents either models verified against other models or present measurement techniques used to measure attenuation. No direct comparison between a model and experimental data has been previously presented. This paper proposes an improved model of plume signal attenuation by separately modeling the most significant physical effects causing signal attenuation and later integrating them. First, the chemical equilibrium reaction taking place in the combustion chamber is modeled. Second, a computational fluid dynamics (CFD) model that predicts the expansion of the plume gases as well as the mixing and entrainment of air downstream from the nozzle is presented. Finally, a plasma model that estimates an electromagnetic absorption coefficient is used to estimate signal attenuation through a subscale experimental rocket exhaust plume. A novel approach to collect signal attenuation data from an experiment to verify the proposed model is also presented.

II. Experimental Setup

To validate the proposed LOS-LCF model, signal attenuation through a small rocket exhaust plume is measured and compared to model-based simulations. The rocket plume is generated using the GISS-1 spark igniter from Masten Space Systems [8], consisting of the combustion chamber and nozzle, spark plug, fuel and oxidizer valves, and the ignition detection as shown in Fig. 1. The igniter was developed for main-chamber ignition of liquid-propellant rocket engines, and it uses isopropyl alcohol (IPA) as fuel and gaseous oxygen (GOX) as oxidizer. The igniter setup consists of the igniter body, which includes the ignition chamber and nozzle, propellant feed solenoid valves and associated plumbing, and a spark driver system. For safety purposes, the GISS-1 ignition system also includes an ignition detection system, which monitors combustion chamber backpressure. The GOX is contained in a compressed gas cylinder with a 2758 kPa pressure regulator, with a pressure relief valve included for safety. The GOX is fed to the ignition chamber through an electrically controlled solenoid valve. The liquid IPA is contained in a tank, pressurized using gaseous nitrogen at 2758 kPa. The fuel-oxidizer mixture is ignited in the combustion chamber using a NGK CMR-7A spark plug. The ignition timing sequence is implemented in National Instruments (NI) LabVIEW software.

To benchmark the proposed model with experimental measurements, a novel approach is chosen to measure the attenuation of microwave signals through the exhaust plume. Instead of using an RF transmitter and a receiver antenna on both sides of the plume, the plume is fired through an orifice in the side wall of a rectangular waveguide. For the waveguide used during our measurements (WR62), the only propagating mode for the frequency range 12.4 to 18 GHz is the TE10 (transverse electric) mode. A thermal analysis was performed on the waveguide to determine the transient temperature load on the waveguide when interacting with the hot exhaust plume. A schematic of the experimental setup is shown in Fig. 1.

Both the ignition sequence of the GISS-1 and the measurement sequence in the network analyzer are automated via NI LabVIEW. The network analyzer is controlled through General Purpose Interface Bus: LabVIEW opens and closes the solenoid valves for both GOX and IPA, and switches the spark igniter on and off. The LabVIEW program ensures that ignition is detected, using the pressure switch in the ignition detection unit before it starts the attenuation measurements.

An Agilent 8720D network analyzer is connected to both sides of the WR62 waveguide. The transmitting port of the network analyzer supplies constant power (−10.0 dBm) at a constant frequency between 12.4 and 18 GHz. The power travels through the waveguide and is received at the other port of the network analyzer, where the received power is measured. The dielectric change inside the waveguide due to the presence or absence of the rocket exhaust plume causes a difference between transmitted and received power, which determines the attenuation. The frequency range, from 12.4 to 18 GHz, is divided in 0.1 GHz intervals, resulting in 57 measurements. Each measurement lasts 15 s, including 2 s of igniter firing, taking 100 measurements per second.

Three measurement sequences are shown in Fig. 2, at 13.5 GHz (Fig. 2a), 15.5 GHz (Fig. 2b), and 17.5 GHz (Fig. 2c). The measurements are 15 s long, with the rocket plume firing between 5
and 7 s. During firing, an increase in attenuation can be clearly seen. The attenuation due to the exhaust plume is calculated by subtracting the received power during firing from the received power without the plume present. This gives values of 0.102, 0.089, and 0.041 dB for 13.5 GHz in Fig. 2a; 15.5 GHz in Fig. 2b; and 17.5 GHz in Fig. 2c, respectively.

The measurement includes time before and after firing in order to identify the trend caused by heat radiation due to the plume exhaust, which heats up the air inside the waveguide. Heating causes a change in the electric permittivity of the air inside the waveguide, and therefore a change in the energy transmission through the waveguide, as shown in Fig. 2c. The experimental data are used in two ways. First, the experiment is used to validate the CFD and combustion calculation performed in Sec. III. Second, the data collected from the experiment are also used to verify the RF signal attenuation model presented in Secs. IV and V.

### III. CFD Model of the Exhaust Plume

The purpose of this section is to develop a detailed model of the rocket plume’s structure and chemistry, which is later used in the proposed LOS-LCF attenuation model. Most papers in the existing literature do not make use of a detailed model of the plume flow and composition, and instead rely on either a single-slab model, in which the plasma is treated as uniform at constant $N_e$ and $v_e$, or a stacked-slab approach, which treats the inhomogeneous plasma as a series of parallel layers of arbitrary thickness, arbitrary $N_e$ values, and arbitrary $v_e$ values [9]. A single-slab or stacked-slab approach assumes a uniform temperature and pressure across the slab, and species concentrations are derived from these slab parameters. The major benefit of using a CFD model is obtaining better knowledge of plume structure, including entrainment of air and shear mixing of ambient air and exhaust plume. This information cannot be obtained from a single-slab or stacked-slab approach. Also, the CFD model employs a detailed grid of nodes, and evaluates temperature, pressure, and species concentration at each node. This gives a finer discretization than a single- or stacked-slab approach, in both radial and axial directions of the nozzle exit. Furthermore, CFD can capture the expansion characteristics of an underexpanded flow, as well as the compressive characteristics of an overexpanded jet, into the ambient atmosphere, whereas the slab model assumes an ideal shape regardless of ambient pressure. This section demonstrates how CFD simulations are used to generate temperature, pressure, and chemical species fields for use in the proposed attenuation model.

The chemical structure of a rocket’s plume depends largely on the fuel and oxidizer combination that is used to produce thrust. Hydrogen–oxygen combustion products remain near the equilibrium composition at the local temperature and pressure during expansion through the nozzle into the ambient atmosphere. In contrast, kerosene and oxygen combustion products are better characterized as frozen flow. In this work, these two limiting cases are modeled using chemical equilibrium calculations to determine the plume’s composition as a function of the static temperature and pressure, as well as distance from the nozzle exit. Future work will add reaction-rate chemistry models to the CFD, to more accurately model the physics of a nonequilibrium expansion process. The deviation of equilibrium assumption during an expansion process depends on how the reaction time scale compares with the characteristic expansion time [10].

The high-velocity exhaust gases that make up a rocket plume also entrain large volumes of ambient air. The entrainment of air and mixing of combustion gases with ambient air gives rise to secondary reactions as the high temperature combustion gases react with oxygen in the air to form a multitude of product species. Near the nozzle exit, the combustion products are not yet in contact with the ambient air, and reactions within the plume are limited to equilibrium and nonequilibrium species changes due to the changing temperature and pressure fields. This zone is suitable for making measurements of static thrust and for comparison with the non-reactive CFD simulation.

There are numerous examples of detailed CFD simulations of the combustion process and chemical kinetics of rocket combustion chambers, nozzles, and exhaust plumes. Chenoweth et al. [11] discussed a CFD tool that simulates the primary breakup of liquid fuel injection spray into droplet, droplet vaporization, droplet combustion, and nonequilibrium calculations. The chemical kinetics and afterburning effect downstream of the nozzle were thoroughly examined in [12], where the gas properties at the nozzle exit plane are used as input and boundary conditions for a dedicated rocket plume simulation code that takes into consideration the nonequilibrium and afterburning effects downstream of the nozzle. In another study [13], the afterburning effects downstream of the nozzle of a solid rocket booster in the stratosphere are simulated. It is shown that ozone in the stratosphere increases the afterburning effects in a rocket plume, and new chemical species are generated. The ion-particle distribution and electron density in the plume are shown to vary with propellant composition and flight altitude. Finally, Victor [3] refers to ionic species as minor but critically important for accurate simulation of the exhaust plume. The rocket exhaust plume model used in this paper neglects ionic species. It will be shown in Sec. IV that collision frequency is dominated by neutral species, and electron density cannot be predicted accurately assuming frozen flow or chemical equilibrium.

The NASA Chemical Equilibrium with Applications (CEA) code is used to calculate the chemical composition of the exhaust plumes in the experiments described in Sec. II. CEA** calculates chemical equilibrium product concentrations for many combinations of reactants, and determines thermodynamic and transport properties of the product mixture. Based on the input conditions, CEA is used to calculate the equilibrium reactions that take place in the chamber region. The plume is modeled as either a single-slab or stacked-slab approach. Also, the CFD model employs a detailed grid of nodes, and evaluates temperature, pressure, and species concentration at each node.

*Data available online at [http://www.grc.nasa.gov/WWW/CEAWEB/ceaHome.htm](http://www.grc.nasa.gov/WWW/CEAWEB/ceaHome.htm) [retrieved 26 October 2010].*
expansion process, or a shifting equilibrium calculation where the species are updated based on the local temperature and pressure. The actual experimental conditions are somewhere between these two limiting cases, and the combination of chemical kinetics with a CFD tool allows us to model the evolution of the nonequilibrium species.

For this study, the oxidizer-to-fuel (O/F) ratio is 0.715, and the chamber pressure is 413.69 kPa. The CEA equilibrium calculation estimates a temperature of 1345 K in the chamber, and chemical species of the products with the corresponding mole fractions are given in Table 1.

Using the output given by CEA, a density-based implicit species model was simulated in CFD using second-order discretization for flow and turbulence parameters. The chamber pressure was 4.1 atm (414 kPa or 60 psi) and chamber temperature was 1345 K (1961 °F). A two-dimensional axisymmetric model of the chamber, nozzle, and flowfield downstream of the nozzle was created in Gambit (a preprocessor for Fluent) and a grid resolution study was performed. Figure 3a shows the schematic of domain and boundary conditions for the igniter. The dimensions of the igniter geometry for CFD modeling were obtained from the CAD drawing of GIS2-1. The chamber length is 52.2 mm, chamber diameter is 8.4 mm, throat length is 8.2 mm, and throat diameter is 3.9 mm. The contraction angle from the chamber-to-throat region is 118 deg. Ambient air domain (reservoir) length for the CFD model is 386 mm and reservoir diameter is 356 mm. The chamber is modeled via a pressure inlet boundary condition and the inputs for the inlet boundary condition are pressure, temperature, and species concentration. The chamber domain and nozzle geometry are modeled as adiabatic wall boundary conditions. Flowfield downstream of the nozzle is patched with ambient air at sea-level conditions and the domain extent of ambient air is the pressure outlet. The simulation presented here is for a nonreactive species model (frozen flow assumption) and was modeled using CFD code Fluent version 6.3. The ideal gas law is given in Table 1.

The temperature profile from the CFD simulation is used to more accurately represent the plume structure, avoiding simplifications such as in the stacked-slab plasma model. Second, an improved beam-path calculation through the plume is used, where the plasma properties along the path are used to calculate the radio-wave attenuation. Figure 4 shows a flowchart that describes the proposed LOS-LCF attenuation model.

The attenuation model requires knowledge of the electron-neutral body-collision frequency and the plasma frequency. Some plasma properties need to be known before these two quantities can be determined. The plasma state is a mixture of electrons, ions, and neutral particles moving in random directions that, on average, is electrically neutral \( n_e = n_i \). These free-charge carriers make plasmas electrically conducting. Not all particles in a plasma are ionized, and partial ionization is often assumed. The plasma state is characterized by its corresponding particle densities. When the density of charged particles is low, the neutral particles dominate the interaction between charged particles; that is, the collision process. The temperature difference between electrons and heavy neutral particles is proportional to the square of the ratio of the energy an electron receives from the electric field \( E \) to the pressure \( p \) [14]. Only for small values of \( E/p \) do the temperatures of electrons and heavy particles approach each other, a basic requirement for local

\[
\text{Species} \quad \frac{\text{Mole fraction}}{\text{IPA}} \quad \frac{\text{Mole fraction}}{\text{O}} \quad \frac{\text{Mole fraction}}{\text{CO}} \quad \frac{\text{Mole fraction}}{\text{CO}_2} \quad \frac{\text{Mole fraction}}{\text{H}_2} \quad \frac{\text{Mole fraction}}{\text{H}_2\text{O}} \quad \frac{\text{Mole fraction}}{\text{Other}}
\]

<table>
<thead>
<tr>
<th>Species</th>
<th>O/F = 0.715 and P = 413.69 kPa using CEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPA</td>
<td>0.000000</td>
</tr>
<tr>
<td>O</td>
<td>0.000000</td>
</tr>
<tr>
<td>CH₄</td>
<td>0.000068</td>
</tr>
<tr>
<td>CO</td>
<td>0.39976</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.02871</td>
</tr>
<tr>
<td>H₂</td>
<td>0.50080</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.07004</td>
</tr>
<tr>
<td>Other</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
thermodynamic equilibrium (LTE) in a plasma. LTE requires chemical equilibrium as well as restrictions on the particle concentration gradients. When these conditions are met, the plasma is called thermal plasma. In situations of large departure from these conditions, the plasma is called a nonequilibrium plasma or nonthermal plasma.

The next sections discuss the details of the proposed LOS-LCF attenuation model as outlined in the flowchart shown in Fig. 4. The cross section for momentum transfer is discussed in Sec. IV.A, and the results are used in Sec. IV.B to derive the electron-neutral body collision frequency. The electron density is described in Sec. IV.C, and the results are used in Sec. IV.D to calculate the plasma frequency. The LOS-LCF attenuation model predicts the microwave signal attenuation through a plasma based on a formula for the wave propagation constant $k$. This constant is derived by applying Maxwell’s equations to the transmission of a radio wave through a plasma. The microwave attenuation through a plume is then calculated as the loss of energy per unit distance traveled by the radio signal. The theory used to calculate signal attenuation is presented in Sec. IV.E, which results in a formula to predict the energy loss per unit distance traveled by the radio signal. The cross section for elastic collisions is related to kinetic energy and momentum transfer between colliding partners, hence its importance role in physical kinetics, plasma conductivity, drift, and diffusion and absorption of electromagnetic energy [15].

The electron collision MTCS represents the probability of elastic collision between an electron and a neutral species in the plume. Victor [3] gives simplified equations to calculate MTCS values $Q_{n}$ as a function of electron velocity $v_{e}$ for the species of interest. The mean thermal electron velocity $v_{e}$, in cm/s, is defined by McMillon [15] as

$$v_{e} = \sqrt{\frac{8k_{B}T}{\pi m_{e}}}$$

Comparing the simplified equations to published values of measured MTCS show that the simplified equations can only be used for a small range of electron energy (eV) [3,15]. The mean electron velocity does not account for the fact that an exhaust plume is a nonthermal plasma,
and therefore the electron temperature is higher than the temperature of the neutral species in the plasma.

Published measured values of MTCS have been used for the estimation of the collision cross section for momentum transfer. MTCS data for electron collisions with carbon monoxide molecules are given in [15] for the low-energy range (less than 0.5 eV), and in [16] for the high-energy range (greater than 0.5 eV). The data for methane molecules are given in [17], for carbon dioxide molecules in [18], hydrogen molecules in [19], and water molecules in [20].

B. Electron-Neutral Body-Collision Frequency

As mentioned in Sec. IV.A, elastic collisions are the main contributor to electromagnetic absorption. This scattering occurs between electrons and the uncharged atoms and molecules. The electron-neutral body collision frequency \( \gamma \) is the average number of collisions between electrons and atoms or molecules per unit time, and is defined by McMillion [15] and given by Eq. (1).

\[
\gamma = \frac{v_e}{\tau_e} \sum_{i=1}^{N_i} (N_i q_i)
\]

where the mean thermal electron velocity \( v_e \) is defined in Eq. (1).

C. Electron Density

Electron density is the probability of an electron being present at a specific location. For most rocket engines, temperatures at the exit plane are so low that little ionization is expected under the assumption of thermodynamic equilibrium. However, experiments show that the electron density in the exhaust of large-scale motors is significant [21]. The electron density is composed of both mean (time-averaged) and fluctuating components. In radar signals, the mean values are required to calculate signal attenuation, reflection, and phase shift, whereas the fluctuating components are used for determining the radar beam's cross section [22].

To predict electron density, a nonequilibrium chemical analysis of both combustion and plume is needed. This explains why previous LOS models are inaccurate in predicting signal attenuation in rocket exhaust plumes with more than 5% aluminum. Smoot and Underwood [2] conclude that the presence of aluminum increases the electron density significantly. The distribution of electrons cannot be assumed to follow a Boltzmann distribution because the rocket exhaust plume is nonthermal: pressure, temperature, and particle density have more significant influence on electron density. Smoot and Underwood [2] show that doubling the chamber pressure results in an increase of approximately 28% in electron density. Temperature dependence is hard to define because it influences the reaction mechanisms of each species differently. This is not considered in the proposed plume model because the density of air and species at the measurement point are approximately constant.

Electron density in the improved LOS-LCF model is obtained from the temperature and pressure field of the igniter plume, obtained from the CFD solution discussed in Sec. III. The CFD solution is exported into Matlab, each grid node location within the plume is identified, and the associated temperature and pressure for each node is recorded. The electron density is curve fitted based on measurement data combined with the temperature, pressure, and species mole fractions obtained from the CFD simulation.

Besides [2], none of the studies found in the literature give predicted or measured values for electron density. It is not possible to measure absolute electron density inside of a high-temperature exhaust. A Langmuir probe is only able to measure differences in electron densities outside the plume. Therefore, electron density in the plume has been curve fitted from measured data.

D. Plasma Frequency

The plasma frequency describes the frequency at which the electron density in the plasma oscillates. The plasma frequency, also known as electron plasma frequency [Eq. (3)], is the most fundamental time scale in plasma physics. There is a different plasma frequency for each species, and the relatively fast electron frequency is the most important:

\[
a_0^p = \frac{\gamma_0 / \sigma_e}{\varepsilon_0 m_e} = \frac{v_e}{\tau_e} \sum_{i=1}^{N_i} (N_i q_i)
\]

If the plasma conductivity is low (\( \sigma_e \ll \omega \varepsilon_0 \mid \delta t \mid \)) and the frequency is sufficiently high (\( \omega \gg \sigma_e \)), electromagnetic waves propagate easily in a plasma. When the frequency decreases, the dielectric permittivity becomes negative and total reflection occurs. This occurs at the critical electron density [15] given by

\[
n_{e}^{\text{crit}} = \frac{\varepsilon_0 m_e \omega^2}{e^2}
\]

E. Line-of-Sight Attenuation Model

To obtain an attenuation model, an investigation of wave propagation through a plasma is required. Combining two Maxwell’s equations (Ampère’s circuital law and the Maxwell–Faraday equation) results in the dispersion relation. By solving the dispersion relation, an expression for the wave number \( k \) is obtained. This solution is used in the collision model analysis, which starts with the Langevin force equation and results in the collision dispersion relation. A planar electromagnetic wave propagating through a conducting medium is governed by Maxwell’s equations:

\[
\nabla \times \vec{B} = \mu_0 \vec{J} + \sigma_0 \frac{\partial \vec{E}}{\partial t} \quad (5)
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (6)
\]

Combining the Maxwell–Faraday equation with Ampère’s law yields the wave equation:

\[
\nabla \times (\nabla \times \vec{E}) + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \nabla \left( \frac{\mu_0}{\varepsilon_0} \nabla \times \vec{J} \right) = 0 \quad (7)
\]

Assuming linearity, the wave fields are sufficiently small, and so current is a linear function of the electric field. Assuming the medium is homogeneous in space and time makes it possible to analyze the fields and currents by Fourier

\[
\vec{E}(\mathbf{r}, t) = \int \vec{E}(k, \omega) \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega t) \quad (8)
\]

and write an Ohm’s law expression for each Fourier mode,

\[
\vec{J}(k, \omega) = \sigma(k, \omega) \cdot \vec{E}(k, \omega) \quad (9)
\]

where \( \sigma \) is the conductivity tensor. In general, a plasma does not satisfy the spatial uniformity assumption. However, if gradients are sufficiently small, then this analysis can be applied locally. An Eikonal or geometrical optics approximation can be used. For a single Fourier mode, the wave equation yields

\[
\nabla \times (\nabla \times \vec{E}) + \frac{j \omega}{c} \vec{J} + \frac{\mu_0}{\varepsilon_0} \nabla \times \vec{J} = 0 \quad (10)
\]

\[
(\nabla \times \vec{E}) \cdot \vec{E} = 0 \quad (11)
\]

Where \( \varepsilon \) is the permittivity tensor and \( I \) is the unit dyad:

\[
\varepsilon = \begin{pmatrix} 
\varepsilon_0 & j \frac{\sigma}{\omega} \\
-j \frac{\sigma}{\omega} & \varepsilon_0
\end{pmatrix}
\]

The plasma properties may be equivalently specified by \( \varepsilon \) or \( \sigma \). A nontrivial solution for \( \vec{E} \) is
det\( (k^2 - k^2 I + \omega^2 \mu_0 \varepsilon) = 0 \) \hspace{1cm} (13)

which is the dispersion relation, relating \( k \) and \( \omega \) for different waves. Its solution also gives the wave polarization. When the plasma is not immersed in a magnetic field, it can be treated as an isotropic medium. Then,

\[ \sigma = \sigma I \] \hspace{1cm} (14)

\[ \varepsilon = \varepsilon I \] \hspace{1cm} (15)

where \( \sigma \) and \( \varepsilon \) are the scalar conductivity and permittivity. The dispersion relation in Eq. (13) reduces to

\[ (-k^2 + \omega^2 \mu_0 \varepsilon)^2(\omega^2 \mu_0 \varepsilon) = 0 \] \hspace{1cm} (16)

Solving the simplified dispersion relation gives two solutions:

\[ \omega^2 \mu_0 \varepsilon - k^2 = 0 \] \hspace{1cm} (17)

or

\[ \omega^2 \mu_0 \varepsilon = 0 \] \hspace{1cm} (18)

Equation (17) represents a solution with transverse polarization of the electric field \( (k \cdot \vec{E} = 0) \), whereas Eq. (18) gives the longitudinal solution \( (\vec{k} \times \vec{E} = 0) \). The transverse solution gives an expression for the wave number \( k \):

\[ k = \frac{\omega}{c} \sqrt{\varepsilon_r} \] \hspace{1cm} (19)

The longitudinal wave dispersion relation gives \( \varepsilon = 0 \), which has no physical meaning and can, therefore, be ignored. The average electron drift velocity is related to the electric field by Langevin’s force equation:

\[ m_e \frac{dv_e}{dt} = -e\vec{E} - \gamma m_e v_e \] \hspace{1cm} (20)

Applying Fourier analysis to Eq. (20) yields

\[ v_e = \frac{e}{m_e(\gamma - j\omega)} \hat{E} \] \hspace{1cm} (21)

Using \( \hat{E} = -en_e v_e = \sigma \hat{E} \) gives

\[ \sigma = \frac{e^2 n_e}{m_e(\gamma - j\omega)} \] \hspace{1cm} (22)

The permittivity is given by

\[ \varepsilon = \varepsilon_0 + \frac{1}{\omega} \sigma \] \hspace{1cm} (23)

which gives

\[ \varepsilon_r = 1 - \frac{\omega_0^2}{\omega(\gamma + \omega)} \] \hspace{1cm} (24)

or

\[ \varepsilon_r = \left( 1 - \frac{\omega_0^2}{\gamma^2 + \omega^2} \right) + j \left( \frac{\gamma \omega_0^2}{\omega(\gamma^2 + \omega^2)} \right) \] \hspace{1cm} (25)

Using the transverse solution found in Eq. (19) gives the collision dispersion relation:

\[ k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_0^2}{\gamma^2 + \omega^2} + j \frac{\gamma \omega_0^2}{\omega(\gamma^2 + \omega^2)}} \] \hspace{1cm} (26)

Introducing

\[ A = \frac{\omega_0^2}{\gamma^2 + \omega^2} \] \hspace{1cm} (27)

gives

\[ k = \frac{\omega}{c} \sqrt{1 - \frac{1}{2}(1 - A) + j \frac{\gamma A}{\omega^2}} \] \hspace{1cm} (28)

Assuming \( k = (j\alpha - \beta) \), with \( \alpha \) as the attenuation constant and \( \beta \) as the phase constant, yields

\[ \alpha = \frac{\omega}{c} \sqrt{-\frac{1}{2} + \frac{1}{2} \sqrt{1 - (1 - A)^2 + \frac{\gamma^2}{\omega^2} A^2}} \] \hspace{1cm} (29)

\[ \beta = -\frac{\omega}{c} \sqrt{-\frac{1}{2} + \frac{1}{2} \sqrt{1 - (1 - A)^2 + \frac{\gamma^2}{\omega^2} A^2}} \] \hspace{1cm} (30)

F. Assessment of Microwave Propagation

To verify the requirement that “the transverse focused interaction of plume and beam axis for a beam half-power radius should be no more than one-fourth the plume radius” [23], the power distribution of the microwave signal is calculated. In the proposed model, the power distribution of the propagating microwave signal is used when calculating the attenuation through the plasma, instead of a single ray, as used in all previous models found. The properties of the microwave inside a waveguide are known and can, therefore, be introduced in the path calculations.

All the models found in the literature consider the microwave path as a single ray. The resulting models are not accurate if the beam is not focused enough.

A rectangular waveguide is an enclosed conductor filled with an insulator. The operating modes on a waveguide are transverse electric (TE) and transverse magnetic (TM) waves. Transverse electromagnetic (TEM) waves are not supported.

First, the fields within the waveguide are evaluated. Assuming the air in the waveguide is homogeneous and the walls are ideal conductors gives the boundary conditions \( E_x, E_z = 0 \) at \( y = 0 \) and \( E_y, E_x = 0 \) at \( x = 0, a \).

Assuming that the behavior for all modes in the \( z \) direction is described by \( \exp(-kz) \gives the phase field representation:

\[ \hat{E} = \hat{E}(x, y)e^{-kz}; \hat{H} = \hat{H}(x, y)e^{-kz} \] \hspace{1cm} (31)

Solving Maxwell’s equations results in the following general description of the wave number \( k \) [24]:

\[ k_{mn} = \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 - \omega^2 \varepsilon_0 \mu_0} \] \hspace{1cm} (32)

The transport of energy and information is described by \( k = \alpha + \beta \), which is a function of the indices \( m \) and \( n \) of the modes in Eq. (32). For low frequencies, \( k = \alpha \) and no transport of energy occurs. A wave propagating in the \( z \) direction occurs at the specified frequency \( k = j\beta \). The waveguide acts as a high-frequency pass filter with a cutoff frequency \( f_{c,mn} \), which is the same for both TE_{mn} and TM_{mn}.

The cutoff frequency is defined as

\[ f_{c,mn} = \frac{1}{2\pi \sqrt{\varepsilon_0 \mu_0} \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2}} \] \hspace{1cm} (33)
Replacing the wave equations from Eq. (34) gives

$$S = \frac{\text{Im}(k_{10})\omega\mu C^2}{(\pi/a)^2} \sin^2 \left( \frac{\pi x}{a} \right)$$

Only the change in received power (with and without the exhaust plume present) is relevant to predict the signal attenuation due to the plume. The average Poynting vector is normalized such that

$$\int_0^a \int_0^b S \, dx \, dz = 1$$

The normalized average Poynting vector is now

$$S_{\text{norm}} = \frac{2}{b} \sin^2 \left( \frac{\pi z}{a} \right)$$

The normalized average Poynting vector is used to weigh each microwave ray that travels through the waveguide. This is done because each ray contains a different amount of power, which will also contribute differently to the total attenuation of the microwave beam. This approach satisfies the focused beam requirement stated at the beginning of this section.

V. Experimental Assessment of Proposed LOS-LCF Model

Frequency attenuation measurements through a hot gas plume composed of IPA-GOX combustion products were performed over a frequency range of 12.4 to 18.0 GHz. The exhaust plume structure was estimated by the expansion of the hot products through a nozzle into an ambient atmosphere. The attenuation measurements were performed multiple times for repeatability at frequency increments of about 0.1 GHz. Sample attenuation results are shown in Fig. 2 at frequencies of 13.5, 15.5, and 17.5 GHz. A summary of two sets of attenuation data, measured at a downstream distance of 2.5 mm from the nozzle exhaust plane, is shown in Fig. 5.

The average attenuation over the full frequency range was 0.0642 dB, and the peak measured attenuation was 0.1124 dB, at a frequency of 13.9 GHz. The standard deviation is 0.0175 and the peak-to-peak differences in measured attenuation level are largely attributable to the relatively short firing duration, which makes it difficult to accurately estimate the average attenuation during firing. Although the data show scatter between 0.03 and 0.11 dB, the trend for both data sets is to show decreasing attenuation with increasing measurement frequency, and the decreasing trend is also present when using worst-case estimates for the data uncertainty. The decreasing-attenuation-with-increasing-frequency trend may be explained using the theory described in the Sec. IV: the electron

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cutoff frequency, GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE_{10}</td>
<td>10.07</td>
</tr>
<tr>
<td>TE_{01}</td>
<td>20.15</td>
</tr>
<tr>
<td>TE_{20}</td>
<td>20.15</td>
</tr>
<tr>
<td>TE_{11}</td>
<td>22.52</td>
</tr>
<tr>
<td>TM_{11}</td>
<td>28.49</td>
</tr>
<tr>
<td>TE_{21}</td>
<td>28.49</td>
</tr>
<tr>
<td>TM_{21}</td>
<td>28.49</td>
</tr>
<tr>
<td>TE_{01}</td>
<td>30.22</td>
</tr>
</tbody>
</table>

The restrictions on the indices $m$ and $n$ for TE_{mn} and TM_{mn} modes are different. For TE_{mn}, one of the indices can be zero, whereas for TM_{mn}, neither of the indices can be zero. Using Eq. (33) with the dimensions of the WR62 waveguide used in our experiments ($a = 0.01580$ m and $b = 0.00790$ m), TE_{mn} and TM_{mn} give the cutoff frequencies shown in Table 2.

The frequency range specified for waveguide WR62 is 12.4 to 18 GHz, which allows only the dominant mode to propagate. This is the only mode taken into account when predicting the electric and magnetic fields inside the waveguide. The equations for TE_{10} are given by

$$E_x = 0 \quad E_z = -\frac{\omega \mu C}{(\pi/a)^2} a \sin \left( \frac{\pi x}{a} \right) e^{-k_{10} z} \quad E_z = 0$$

$$H_x = \frac{k_{10} C}{(\pi/a)^2} a \sin \left( \frac{\pi x}{a} \right) e^{-k_{10} z} \quad H_y = 0$$

$$H_x = C \cos \left( \frac{\pi x}{a} \right) e^{-k_{10} z}$$

with $a = 0.01580$ m, $b = 0.00790$ m,

$$k_{10} = \sqrt{\left( \frac{\pi}{a} \right)^2 - \omega^2 \epsilon \mu}$$

The average Poynting vector is defined as

$$S = \frac{1}{2} \text{Re}(E \times H^*)$$

In a rectangular waveguide, this simplifies to

$$S = \frac{1}{2} \text{Re}(E_x H_z^* - E_z H_x^*)$$

Fig. 5 Comparison of attenuation measurements vs model-predicted attenuation.
density (directly related to the plasma frequency) indicates a plasma frequency smaller than the measurement frequencies used in the experiment, which leads to a decrease in attenuation. Low plume temperature leads to low expected electron density, which results in fewer free electrons.

Attenuation predictions from three different LOS attenuation models are also shown in Fig. 5. The temperature, pressure, and species concentration for each model are obtained from the CFD model of the plume. The simplified MTCS model is based on the collision frequency, calculated using simplified formulas for the cross sections for momentum transfer. Interestingly, the MTCS for CO, CO₂, and H₂ predict a cross section that is approximately 40% lower, and would, therefore, be closer to the expected values based on the measurements. However, the MTCS of H₂O is 105 times higher than the prediction based upon the values found in [20].

The improved LOS model uses the collision frequency calculated based on the CFD exhaust plume model. The electron temperature is taken to be the same as the species temperature. The actual electron temperature is expected to be higher because the exhaust plume is considered a nonthermal plasma. A lower neutral particle density is also expected because the assumption of chemical equilibrium gives a higher concentration of neutral particles than in the nonequilibrium state of the plume. Elevated densities of neutral particles make the collision frequency higher. The estimated cross section can be higher or lower than the actual value depending on species and temperature, and collision frequency is directly dependent on the mean electron velocity [Eq. (4)]. At the plume temperature (1000 K or lower), the total collision frequency will increase when the electron temperature is assumed to be higher.

The proposed LOS-LCF model uses a lower collision frequency than the improved LOS model. A lower collision frequency is expected because the number of neutral particles is lower for nonequilibrium conditions than for the assumed equilibrium in the chamber and frozen flow in the nozzle. The predicted mean electron density in the plume for the proposed LOS-LCF model is \(3.02 \times 10^{17} \text{ m}^{-3}\), which is 30% lower than the mean electron density used in the improved LOS model. These values are close to those shown in [2]. The values seem slightly high considering that the combustion temperature and pressure in the experiment are considerably lower than those found in [2]: combustion temperature is approximately 3 times lower, and pressure is approximately 10 times lower.

Calculating the L2 error norm between each model and the measurement data gives 0.1063 for the first improved LOS model, 0.1013 for the second improved LOS model, and 0.1127 for the benchmark model, which demonstrates that the proposed LOS-LCF model predicts the actual attenuation the best.

In Fig. 6, the predicted losses of these models are plotted over a wider frequency range. This figure is based on simulations. The experiments were performed over a smaller frequency range (shown in red).

The inaccuracy of the simplified MTCS model at higher frequencies is clear: the attenuation remains constant for increasing frequencies. In the other two models, the peak in attenuation is due to total reflection, which occurs when the measurement frequency is the same as the plasma frequency. The shape of the attenuation curve is determined by both the plasma and collision frequencies. The plasma frequency determines the location of the peak, and the collision frequency determines the width of the peak.

The proposed model focuses on prediction of signal power attenuation. For this reason, only the attenuation parameters in the transmission network \(S_{11}\) were measured. Estimation of the phase constant \(\beta\) in Eq. (30) could provide means to verify the electron density estimation by using the network analyzer to measure the average phase shift through the rocket exhaust plume. The phase shift caused by the plume should be equal to the phase constant calculated using this equation.

VI. Conclusions

The improved line-of-sight lower collision frequency (LOS-LCF) model presented in this paper is based on a more accurate description of the microwave path, and improved estimations of the cross section for momentum transfer (MTCS) based on avoiding typical simplifying assumptions in the plume structure, achieved by incorporating a computational fluid dynamics model of the exhaust plume. The improved LOS-LCF model predicts the attenuation of a radio signal through a hot plume of isopropyl alcohol and gaseous oxygen combustion products better than the earlier published simplified MTCS. Other published LOS models are not as adept at capturing the decreasing behavior of the attenuation at higher frequencies. The predicted collision frequency in these models is typically too high because the chemical equilibrium combustion model followed by the nonreacting frozen-flow gas-mixing model predicts more and different neutral species than the nonequilibrium chemical model with afterburning in the atmosphere. Better predictions are achieved with the LOS-LCF model, which makes use of fitted electron density and collision frequency.

The improved LOS-LCF model will allow designers of communication equipment for rockets and ground stations to replace the extremely conservative estimate of plume signal attenuation (typically in the order of 20 dB) commonly used in practice, with a calculated attenuation based on the worst-case path through the plume and the known plume plasma properties. This will allow cost reduction and reduced interference caused by the ground station (due to lower power requirements) as well as an increase in a vehicle’s payload capacity due to mass savings.
Acknowledgments

This project was supported by NASA-Kennedy Space Center through agreement number 09-005 with Analex Corporation. The authors would like to especially thank David Becknell of the Florida Institute of Technology for his extensive help with the construction, testing, and operation of the GIS1-1 spark igniter.

References


M. MacLean
Associate Editor