

**Problem 7.39** Dry soil is characterized by \( \varepsilon_r = 2.5, \mu_r = 1, \) and \( \sigma = 10^{-4} \) (S/m).

At each of the following frequencies, determine if dry soil may be considered a good conductor, a quasi-conductor, or a low-loss dielectric, and then calculate \( \alpha, \beta, \lambda, H, \) and \( \eta_c \):

(a) 60 Hz,
(b) 1 kHz,
(c) 1 MHz,
(d) 1 GHz.

**Solution:** \( \varepsilon_r = 2.5, \mu_r = 1, \) \( \sigma = 10^{-4} \) S/m.

<table>
<thead>
<tr>
<th>( f \rightarrow )</th>
<th>60 Hz</th>
<th>1 kHz</th>
<th>1 MHz</th>
<th>1 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon} )</td>
<td>( \frac{\sigma}{\omega \varepsilon} )</td>
<td>( \frac{\sigma}{2\pi f \varepsilon_r \varepsilon_0} )</td>
<td>1.2 \times 10^4</td>
<td>720</td>
</tr>
<tr>
<td>Type of medium</td>
<td>Good conductor</td>
<td>Good conductor</td>
<td>Quasi-conductor</td>
<td>Low-loss dielectric</td>
</tr>
<tr>
<td>( \alpha ) (Np/m)</td>
<td>( 1.54 \times 10^{-4} )</td>
<td>( 6.28 \times 10^{-4} )</td>
<td>( 1.13 \times 10^{-2} )</td>
<td>( 1.19 \times 10^{-2} )</td>
</tr>
<tr>
<td>( \beta ) (rad/m)</td>
<td>( 1.54 \times 10^{-4} )</td>
<td>( 6.28 \times 10^{-4} )</td>
<td>( 3.49 \times 10^{-2} )</td>
<td>( 33.14 )</td>
</tr>
<tr>
<td>( \lambda ) (m)</td>
<td>( 4.08 \times 10^4 )</td>
<td>( 10^4 )</td>
<td>( 180 )</td>
<td>( 0.19 )</td>
</tr>
<tr>
<td>( \nu_p ) (m/s)</td>
<td>( 2.45 \times 10^6 )</td>
<td>( 10^7 )</td>
<td>( 1.8 \times 10^8 )</td>
<td>( 1.9 \times 10^8 )</td>
</tr>
<tr>
<td>( \eta_c ) (Ω)</td>
<td>( 1.54(1+j) )</td>
<td>( 6.28(1+j) )</td>
<td>( 204.28 + j65.89 )</td>
<td>( 238.27 )</td>
</tr>
</tbody>
</table>

**Problem 7.40** In a medium characterized by \( \varepsilon_r = 9, \mu_r = 1, \) and \( \sigma = 0.1 \) S/m, determine the phase angle by which the magnetic field leads the electric field at 100 MHz.

**Solution:** The phase angle by which the magnetic field leads the electric field is \( -\theta_\eta \) where \( \theta_\eta \) is the phase angle of \( \eta_c \).

\[
\frac{\sigma}{\omega \varepsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^8 \times 10^{-9} \times 9} = 2.
\]

Hence, quasi-conductor.

\[
\eta_c = \sqrt{\frac{\mu}{\varepsilon'}} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-1/2} = \frac{120\pi}{\sqrt{\varepsilon_r}} \left( 1 - j \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^{-1/2} = 125.67(1-j2)^{-1/2} = 71.49 + j44.18 = 84.04 \angle 72^\circ.
\]
Therefore $\theta_H = 31.72^\circ$.

Since $H = (1/\eta_e)k \times E$, $H$ leads $E$ by $-\theta_H$, or by $-31.72^\circ$. In other words, $H$ lags $E$ by $31.72^\circ$.

**Problem 7.41** Generate a plot for the skin depth $\delta_s$ versus frequency for seawater for the range from 1 kHz to 10 GHz (use log-log scales). The constitutive parameters of seawater are $\mu_e = 1$, $\varepsilon_r = 80$ and $\sigma = 4$ S/m.

**Solution:**

\[
\delta_s = \frac{1}{\alpha} = \frac{1}{\omega} \left[ \frac{\mu \varepsilon'}{2} \left[ \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right]^{-1/2}
\]

\[
\omega = 2\pi f,
\]

\[
\mu \varepsilon' = \mu_0 \varepsilon_0 \varepsilon_r = \frac{\varepsilon_r}{c^2} = \frac{80}{(3 \times 10^8)^2} \frac{80}{(3 \times 10^8)^2} = \frac{4 \times 36\pi}{2\pi f \times 10^{-9} \times 80} = \frac{72}{80f} \times 10^9.
\]

See Fig. P7.41 for plot of $\delta_s$ versus frequency.

![Skin depth vs. frequency for seawater](image)

Figure P7.41: Skin depth versus frequency for seawater.
Since $\alpha = 1/\delta_y = 1/5 = 0.2 \text{ Np/m},$

$$\sigma = \frac{\alpha}{19.87} = \frac{0.2}{19.87} = 0.01 \text{ S/m}.$$ 

(b) Since $\alpha = \beta$ for a good conductor, and $\alpha = 0.2$, it follows that $\beta = 0.2$. Therefore,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2} = 10\pi = 31.4 \text{ m}.$$ 

(c) $u_p = f\lambda = 10^6 \times 31.4 = 3.14 \times 10^7 \text{ m/s}.$

**Problem 7.45** The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$E = 2.25e^{-30x} \cos(2\pi \times 10^9 t - 40x) \text{ (V/m)}.$$ 

Obtain the corresponding expression for $H$.

**Solution:** From the given expression for $E$,

$$\omega = 2\pi \times 10^9 \text{ (rad/s)},$$

$$\alpha = 30 \text{ (Np/m)},$$

$$\beta = 40 \text{ (rad/m)}.$$ 

From (7.120a) and (7.120b),

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon' = -\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r = -\frac{\omega^2}{c^2} \varepsilon'_r,$$

$$2\alpha\beta = \omega^2 \mu \varepsilon'' = \frac{\omega^2}{c^2} \varepsilon''_r.$$ 

Using the above values for $\omega$, $\alpha$, and $\beta$, we obtain the following:

$$\varepsilon'_r = 1.6,$$

$$\varepsilon''_r = 5.47.$$ 

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon}} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-1/2}$$

$$= \frac{\eta_0}{\sqrt{\varepsilon'_r}} \left( 1 - j \frac{\varepsilon''_r}{\varepsilon'_r} \right)^{-1/2} = \frac{377}{\sqrt{1.6}} \left( 1 - j \frac{5.47}{1.6} \right)^{-1/2} = 157.9 e^{j36.85^\circ} \text{ (W)}.$$
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\[ \mathbf{E} = \hat{x}25e^{-30x}e^{-j40x}, \]

\[ \mathbf{H} = \frac{1}{\eta_e} \hat{k} \times \mathbf{E} = \frac{1}{157.9} \hat{k} \times \hat{x}25e^{-30x}e^{-j40x} = -\hat{y}0.16e^{-30x}e^{-40x}e^{-j36.85\circ}, \]

\[ \mathbf{H} = \Re\{\mathbf{H}_e^{j\omega t}\} = -\hat{y}0.16e^{-30x}\cos(2\pi \times 10^9t - 40x - 36.85\circ) \text{ (A/m)}. \]

Section 7-6: Current Flow in Conductors

Problem 7.46 In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor

\[ \mathbf{H} = (\hat{x} - j4\hat{z})e^{-2y}e^{-j9\circ} \text{ (A/m)}. \]

Obtain time-domain expressions for the electric and magnetic field vectors.

Solution:

\[ \mathbf{E} = -\eta_e \hat{k} \times \mathbf{H}. \]

To find \( \eta_e \), we need \( \varepsilon' \) and \( \varepsilon'' \). From the given expression for \( \mathbf{H} \),

\[ \alpha = 2 \text{ (Np/m),} \]
\[ \beta = 9 \text{ (rad/m).} \]

Also, we are given that \( f = 300 \text{ MHz} = 3 \times 10^8 \text{ Hz} \). From (7.120a),

\[ \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon', \]

\[ 4 - 81 = -(2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon' \times \frac{10^{-9}}{36\pi}, \]

whose solution gives

\[ \varepsilon' = 1.95. \]

Similarly, from (7.120b),

\[ 2\alpha \beta = \omega^2 \mu \varepsilon'', \]

\[ 2 \times 2 \times 9 = (2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon'' \times \frac{10^{-9}}{36\pi}, \]

which gives

\[ \varepsilon'' = 0.91. \]
\[ \eta_e = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - \frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \]
\[ = \frac{\eta_0}{\sqrt{\varepsilon'_r}} \left(1 - \frac{0.91}{1.95}\right)^{-1/2} = \frac{377}{\sqrt{1.95}} \approx \frac{0.93 + j0.21}{256.9 e^{j12.6^\circ}}. \]

Hence,
\[ \tilde{E} = -256.9 e^{j12.6^\circ} \hat{y} \times (\hat{x} - j4\hat{z}) e^{-2y} e^{-j9y} \]
\[ = (\hat{x} j4 + \hat{z}) 256.9 e^{-2y} e^{-j9y} e^{j12.6^\circ} \]
\[ = (\hat{x} 4e^{j\pi/2} + \hat{z}) 256.9 e^{-2y} e^{-j9y} e^{j12.6^\circ}, \]
\[ E = \Re\{\tilde{E} e^{j\omega t}\} \]
\[ = \hat{x} 1.03 \times 10^3 e^{-2y} \cos(\omega t - 9y + 102.6^\circ) \]
\[ + \hat{z} 256.9 e^{-2y} \cos(\omega t - 9y + 12.6^\circ) \quad (V/m), \]
\[ H = \Re\{\tilde{H} e^{j\omega t}\} \]
\[ = \Re\{(\hat{x} + j4\hat{z}) e^{-2y} e^{-j9y} e^{j\omega t}\} \]
\[ = \hat{x} e^{-2y} \cos(\omega t - 9y) + \hat{z} 4e^{-2y} \sin(\omega t - 9y) \quad (A/m). \]

**Problem 7.47** A rectangular copper block is 30 cm in height (along \( z \)). In response to a wave incident upon the block from above, a current is induced in the block in the positive \( x \)-direction. Determine the ratio of the a-c resistance of the block to its d-c resistance at 1 kHz. The relevant properties of copper are given in Appendix B.

![Figure P7.47: Copper block of Problem 7.47.](image-url)
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Solution:

\[ \text{d-c resistance } R_{dc} = \frac{l}{\sigma A} = \frac{l}{0.3 \sigma w}, \]
\[ \text{a-c resistance } R_{ac} = \frac{l}{\sigma w \delta_s}. \]

\[ \frac{R_{ac}}{R_{dc}} = \frac{0.3}{\delta_s} = 0.3 \sqrt{\pi f \mu \sigma} = 0.3 [\pi \times 10^3 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{1/2} = 143.55. \]

**Problem 7.48** The inner and outer conductors of a coaxial cable have radii of 0.5 cm and 1 cm, respectively. The conductors are made of copper with \( \varepsilon = 1, \mu = 1 \) and \( \sigma = 5.8 \times 10^7 \text{ S/m} \), and the outer conductor is 0.1 cm thick. At 10 MHz:

(a) Are the conductors thick enough to be considered infinitely thick so far as the flow of current through them is concerned?

(b) Determine the surface resistance \( R_s \).

(c) Determine the a-c resistance per unit length of the cable.

**Solution:**

(a) From Eqs. (7.127) and (7.132a),
\[ \delta_s = [\pi f \mu \sigma]^{-1/2} = [\pi \times 10^7 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{-1/2} = 0.021 \text{ mm}. \]

Hence,
\[ \frac{d}{\delta_s} = \frac{0.1 \text{ cm}}{0.021 \text{ mm}} \approx 50. \]

Hence, conductor is plenty thick.

(b) From Eq. (7.147a),
\[ R_s = \frac{1}{\delta_s} = \frac{1}{5.8 \times 10^7 \times 2.1 \times 10^{-5}} = 8.2 \times 10^{-4} \Omega. \]

(c) From Eq. (7.151),
\[ R' = \frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right) = \frac{8.2 \times 10^{-4}}{2\pi} \left( \frac{1}{5 \times 10^{-3}} + \frac{1}{10^{-2}} \right) = 0.039 \text{ (\Omega/m)}. \]
Section 7-7: EM Power Density

**Problem 7.49** The magnetic field of a plane wave traveling in air is given by 
\[ \mathbf{H} = \hat{z} 25 \sin(2\pi \times 10^7 t - ky) \text{ (mA/m)} \]. Determine the average power density carried by the wave.

**Solution:**
\[
\mathbf{H} = \hat{z} 25 \sin(2\pi \times 10^7 t - ky) \text{ (mA/m)},
\]
\[
\mathbf{E} = -\eta_0 \hat{y} \times \mathbf{H} = \hat{z} \eta_0 25 \sin(2\pi \times 10^7 t - ky) \text{ (mV/m)},
\]
\[
S_{\text{av}} = \left( \hat{z} \times \hat{x} \right) \frac{\eta_0 (25)^2}{2} \times 10^{-6} = \frac{120\pi}{2} (25)^2 \times 10^{-6} = \hat{y} 0.12 \text{ (W/m}^2). \]

**Problem 7.50** A wave traveling in a nonmagnetic medium with \( \varepsilon_r = 9 \) is characterized by an electric field given by
\[
\mathbf{E} = \left[ \hat{y} 3 \cos(\pi \times 10^7 t + kx) - \hat{z} 2 \cos(\pi \times 10^7 t + kx) \right] \text{ (V/m)}. \]

Determine the direction of wave travel and the average power density carried by the wave.

**Solution:**
\[
\eta \simeq \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \text{ (Ω)}. \]

The wave is traveling in the negative \( x \)-direction.
\[
S_{\text{av}} = -\hat{x} \frac{[3^2 + 2^2]}{2\eta} = -\hat{x} \frac{13}{2 \times 40\pi} = -\hat{x} 0.05 \text{ (W/m}^2). \]

**Problem 7.51** The electric-field phasor of a uniform plane wave traveling downward in water is given by
\[
\mathbf{E} = \hat{x} 10e^{-0.2z}e^{-j0.2\pi} \text{ (V/m)}, \]
where \( \hat{z} \) is the downward direction and \( z = 0 \) is the water surface. If \( \sigma = 4 \text{ S/m} \),

(a) obtain an expression for the average power density,

(b) determine the attenuation rate, and

(c) determine the depth at which the power density has been reduced by 40 dB.
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Solution:
(a) Since $\alpha = \beta = 0.2$, the medium is a good conductor.

$$\eta_c = (1 + j)^{\alpha/\beta} = (1 + j)^{0.2/0.05} = (1 + j)0.05 = 0.0707 e^{45^\circ} \ (\Omega).$$

From Eq. (7.162),

$$S_{av} = \frac{2|E_0|^2}{2|\eta_c|} e^{-2\alpha z \cos \theta_\eta} = \frac{2}{2 \times 0.0707} e^{-0.4z \cos 45^\circ} = 2500 e^{-0.4z} \ (W/m^2).$$

(b) $A = -8.68 \alpha z = -8.68 \times 0.2z = -1.74z \ (dB)$.

(c) 40 dB is equivalent to $10^{-4}$. Hence,

$$10^{-4} = e^{-2\alpha z} = e^{-0.4z}, \quad \ln(10^{-4}) = -0.4z,$$

or $z = 23.03 \ m$.

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Problem 7.52  The amplitudes of an elliptically polarized plane wave traveling in a lossless, nonmagnetic medium with $\epsilon_r = 4$ are $H_{y0} = 6 \ (mA/m)$ and $H_{x0} = 8 \ (mA/m)$. Determine the average power flowing through an aperture in the $y$-$z$ plane if its area is $20 \ m^2$.

Solution:

$$\eta = \sqrt{\frac{\eta_0}{\epsilon_r}} = \sqrt{\frac{120\pi}{4}} = 60 \pi = 188.5 \ \Omega,$$

$$S_{av} = \frac{\eta}{2} [H_{y0}^2 + H_{x0}^2] = \frac{188.5}{2} [36 + 64] \times 10^{-6} = 9.43 \ (mW/m^2),$$

$$P = S_{av}A = 9.43 \times 10^{-3} \times 20 = 0.19 \ W.$$  

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Problem 7.53  A wave traveling in a lossless, nonmagnetic medium has an electric field amplitude of $24.56 \ V/m$ and an average power density of $4 \ W/m^2$. Determine the phase velocity of the wave.

Solution:

$$S_{av} = \frac{|E_0|^2}{2\eta}, \quad \eta = \frac{|E_0|^2}{2S_{av}},$$

or

$$\eta = \frac{(24.56)^2}{2 \times 4} = 75.4 \ \Omega.$$
But
\[ \eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{75.4}}, \quad \varepsilon_r = \left( \frac{377}{75.4} \right)^2 = 25. \]

Hence,
\[ u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{5} = 6 \times 10^7 \text{ m/s}. \]

**Problem 7.54** At microwave frequencies, the power density considered safe for human exposure is 1 (mW/cm²). A radar radiates a wave with an electric field amplitude \( E \) that decays with distance as \( E(R) = (3,000/R) \) (V/m), where \( R \) is the distance in meters. What is the radius of the unsafe region?

**Solution:**
\[ S_{av} = \frac{|E(R)|^2}{2\eta_0}, \quad 1 \text{ (mW/cm}^2) = 10^{-3} \text{ W/cm}^2 = 10 \text{ W/m}^2, \]
\[ 10 = \left( \frac{3 \times 10^3}{R} \right)^2 \times \frac{1}{2 \times 120\pi} = \frac{1.2 \times 10^4}{R^2}, \]
\[ R = \left( \frac{1.2 \times 10^4}{10} \right)^{1/2} = 34.64 \text{ m}. \]

**Problem 7.55** Consider the imaginary rectangular box shown in Fig. 7-27 (P7.55).

(a) Determine the net power flux \( P(t) \) entering the box due to a plane wave in air given by
\[ E = \hat{x}E_0 \cos(\omega t - ky) \quad (\text{V/m}). \]

(b) Determine the net time-average power entering the box.

**Solution:**

(a) \[ E = \hat{x}E_0 \cos(\omega t - ky), \]
\[ H = -\hat{z} \frac{E_0}{\eta_0} \cos(\omega t - ky). \]
\[ S(t) = E \times H = \hat{y} \frac{E_0^2}{\eta_0} \cos^2(\omega t - ky), \]
\[ P(t) = S(t) A|_{y=0} - S(t) A|_{y=b} = \frac{E_0^2}{\eta_0} ac[\cos^2 \omega t - \cos^2(\omega t - kb)]. \]