Is mathematical reasoning just reasoning about mathematics?

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Outline

Is informal mathematical reasoning just the application to mathematics of methods of reasoning common to other discourse, or is it distinctively mathematical?

Four possible answers:

0. There is no such thing as ‘informal mathematical reasoning’: only formalized reasoning, in which the inferential steps are those admissible within a formal system, can count as mathematical.

1. Informal mathematical reasoning is possible, but must employ exclusively mathematical inferential steps, albeit ones that may be characterized informally.

2. Informal mathematical reasoning is possible, and may employ both exclusively mathematical inferential steps, and inferential steps of more general application.

3. Informal mathematical reasoning is possible, and may be understood purely in terms of inferential steps of general application. No exclusively mathematical inferential steps are required for informal mathematics; that is, such steps may ultimately be reduced to instances of general steps.
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1: Exclusively mathematical steps mandatory
2: Exclusively mathematical steps not mandatory
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It cannot be denied that a complex sequence of interlocked blind guesses and cruel rejections may look much like directed thought, just as Darwinian evolution simulates orthogenesis or design. But we must not be hoodwinked into thinking that it is reasoning, or anything else that we know, that drives us forward to what is unknown. What reasoning does is pull us back. Our guesses are not random, of course, but informed; which means only that they are guesses informed by earlier guesses.

David Miller, ‘Do we reason when we think we reason, or do we think?’ *Learning for Democracy*, 1(3), 2005
0: No such thing

To be sure, [deductive arguments] can provide new subjective knowledge, in the way that mathematical proofs uncannily do. Arguments, it may be conceded, do have an exploratory function, even if what they explore is what is already known, or conjectured, about the world, and not the world itself.

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1: Exclusively mathematical steps mandatory

For false diagrams of geometrical figures are not contentious (for the resulting fallacies conform to the subject of the art)—any more than is any false diagram that may be offered in proof of a truth—e.g. Hippocrates’ figure or the squaring of the circle by means of the lunules. But Bryson’s method of squaring the circle, even if the circle is thereby squared, is still sophistical because it does not conform to the subject in hand. So, then, any merely apparent reasoning about these things is a contentious argument, and any reasoning that merely appears to conform to the subject in hand, even though it be genuine reasoning, is a contentious argument: for it is merely apparent in its conformity to the subject matter, so that it is deceptive and plays foul.

Aristotle, *De Sophisticis Elenchis*, 171b.
Bryson’s method of squaring the circle

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Aristotle, De Sophisticis Elenchis, 171b.
2: Exclusively mathematical steps not mandatory

[H]ow much observation, divination, induction, experimental trial, and verification, causation, too (if that means, as I suppose it must, mounting from phenomena to their reasons or causes of being) have to do with the work of the mathematician

J. J. Sylvester, ‘The study that knows nothing of observation,’ *British Association for the Advancement of Science*, Exeter, 1869.
What is a Toulmin Layout?

(a) Basic Layout

(b) Full Layout

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Argument from Analogy

Argumentation Scheme

Similarity Premise  Generally, case $C_1$ is similar to case $C_2$.

Base Premise  $A$ is true (false) in case $C_1$.

Conclusion  $A$ is true (false) in case $C_2$.

Critical Questions

1. Are there differences between $C_1$ and $C_2$ that would tend to undermine the force of the similarity cited?
2. Is $A$ true (false) in $C_1$?
3. Is there some other case $C_3$ that is also similar to $C_1$, but in which $A$ is false (true)?

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Eventually we found that a squared rectangle was equivalent to an electrical network of unit resistances. The horizontal lines in the squared rectangle correspond to the terminals of the network, and the squares correspond to the wires joining them. The current in a wire is measured by the side-length of the corresponding square, and its direction is downward in the rectangle. The top edge of the rectangle corresponds to the positive pole, the terminal at which the current enters the network. The bottom edge likewise corresponds to the negative pole, the terminal from which the current leaves. The magnitude of the current entering at the positive pole and leaving at the negative can be equated to the length of a horizontal side of the rectangle. The voltage drop from pole to pole then measure the vertical side. We now understood that the basic theory of squared rectangles was that of Kirchoff’s Laws of electrical networks. That was something we could look up in textbooks.

Bill Tutte, 1998, *Graph Theory As I Have Known It.*
3: Exclusively mathematical steps not necessary

[T]he reason why doing mathematics yields the sensation of reasoning about concrete entities ‘in the world’ is that such thought processes are comprised of brain activation patterns that are associated with real world stimuli.

3: Exclusively mathematical steps not necessary

[Mathematical reasoning is already in accord with principles and techniques from informal logic—even if this is unnoticed by the practitioners.]