Fallacies in Mathematics

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Maxwell's *Fallacies in Mathematics*

MISTAKE 'a momentary aberration, a slip in writing, or the misreading of earlier work'

HOWLER 'an error which leads *innocently* to a *correct* result'

FALLACY 'leads by *guile* to a *wrong* but plausible conclusion'

E. A. Maxwell, 1959, *Fallacies in Mathematics*, p. 9.

Aristotle's Fallacies

That some reasonings are genuine, while others seem to be so but are not, is evident. This happens with arguments as also elsewhere, through a certain likeness between the genuine and the sham.

Aristotle, *De Sophisticis Elenchis*, 164a.

Bacon's Juggling Feats

For although in the more gross sort of fallacies it happeneth, as Seneca maketh the comparison well, as in juggling feats, which, though we know not how they are done, yet we know well it is not as it seemeth to be; yet the more subtle sort of them doth not only put a man beside his answer, but doth many times abuse his judgment.

Bacon, 1605, *Advancement of Learning*, p. 131.

Threefold Distinction:

Bacon-Gross	Something seems wrong (and is).
Bacon-Subtle	Everything seems OK (but is not).
Bacon-Surprise	Something seems wrong (but is not).

The Guts of Reality

Physicists like to think they're dealing with reality. Some of them are quite arrogant about it and talk as if they were the only ones with a finger in the belly of the real. They think that mathematicians are just playing games, making up our own rules and playing our own games. But with all their physical theories the possibility still exists that space and time are just Kant's categories of apperception, or that physical objects are nothing but ideas in the mind of God. Who can say for sure? Their physical theories can't rule these possibilities out. But in math things are exactly the way they seem. There's no room, no *logical* room, for deception. I don't have to consider the possibility that maybe seven isn't really a prime, that my mind conditions seven to appear a prime. One doesn't—can't—make the distinction between mathematical appearance and reality, as one can—must—make the distinction between physical appearance and reality. The mathematician can penetrate the essence of his objects in a way the physicist never could, no matter how powerful his theory. We're the ones with our fists deep in the guts of reality.

Rebecca Goldstein, 1983, *The Mind Body Problem*, p. 95

Argument[ation] Schemes

There seems to be general agreement among argumentation theorists that argumentation schemes are principles or rules underlying arguments that legitimate the step from premises to standpoints. They characterize the way that the acceptability of the premise that is explicit in the argumentation is transferred to the standpoint.

Bart Garssen, 1999, 'The Nature of Symptomatic Argumentation', p. 225.

Example: Argument from Analogy

Argument Scheme for Argument from Analogy

- **Similarity Premise**: Generally, case C_1 is similar to case C_2.
- **Base Premise**: A is true (false) in case C_1.
- **Conclusion**: A is true (false) in case C_2.

Critical Questions:

1. Are there differences between C_1 and C_2 that would tend to undermine the force of the similarity cited?
2. Is A true (false) in C_1?
3. Is there some other case C_3 that is also similar to C_1, but in which A is false (true)?

Douglas Walton, 2006, *Fundamentals of Critical Argumentation*, pp. 96 f.

Example: Appeal to Expert Opinion

Argument Scheme for Appeal to Expert Opinion

- **Major Premise**: Source E is an expert in subject domain S containing proposition A.
- **Minor Premise**: E asserts that proposition A (in domain S) is true (false).
- **Conclusion**: A may plausibly be taken to be true (false).

Critical Questions:

1. Expertise Question: How credible is E as an expert source?
2. Field Question: Is E an expert in the field that A is in?
3. Opinion Question: What did E assert that implies A?
4. Trustworthiness Question: Is E personally reliable as a source?
5. Consistency Question: Is A consistent with what other experts assert?

Douglas Walton, 1997, *Appeal to Expert Opinion*, pp. 210, 223.

From Argument Schemes to Fallacies

Two ways in which an argument scheme may be fallacious:

1. If it is invariably bad (for example, quantifier shift, question begging);
2. If it is used inappropriately.

Hence “seems good" may be analysed as “is an instance of a not invariably bad argument scheme".

Applicability to Mathematics

1. Many mathematical fallacies of this type;
2. Are there any of this type?

Inference Packages

Inference packages . . . are psychologically-bundled ways of phenomenologically exploring the effect of several assumptions at once without explicit recognition of what those assumptions are.

Jody Azzouni, 2005, ‘Is there still a Sense in which Mathematics can have Foundations?’, p. 9.

Examples of Inference Packages

- Imagine triangles on a plane.
- Imagine triangles on a sphere.
- Imagine triangles on an ellipsoid.

A Fallacy from Newton

Suppose the product or rectangle AB increased by continual motion: and that the momentaneous increments of the sides A and B are a and b. When the sides A and B were deficient, or lesser by one half of their moments, the rectangle was $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$ *i.e.* $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$. And as soon as the sides A and B are increased by the other two halves of their moments, the rectangle becomes $\overline{A + \frac{1}{2}a} \times \overline{B + \frac{1}{2}b}$ or $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. From the latter rectangle subduct the former, and the remaining difference will be $aB + bA$. Therefore the increment of the rectangle generated by the initial increments a and b is $aB + bA$. *Q.E.D.* But it is plain that the direct and true method to obtain the moment or increment of the rectangle AB, is to take the sides as increased by their whole increments, and so multiply them together $A + a$ by $B + b$, the product whereof $AB + aB + bA + ab$ is the augmented rectangle; whence, if we subduct AB the remainder $aB + bA + ab$ will be the true increment of the rectangle, exceeding that which was obtained by the former illegitimate and indirect method by the quantity ab.

George Berkeley, 1734, *The Analyst*, ¶9.

Summary

- Mathematical reasoning can exhibit fallacies—of a variety of types.
- Mathematical fallacies may be characterized in terms of argument schemes.
- Argument schemes generalize independent accounts of mathematical reasoning, such as inference packages, (and perhaps proof schemes).