MATHEMATICS AND ARGUMENTATION

1

The traditional foci of philosophy of mathematics have been the foundations of mathematics and the status of mathematical proof. Its traditional methodology has been the application of formal logic. However, as an increasing number of philosophers of mathematics have complained, these two concentrations omit much of mathematical practice, including much that is of philosophical relevance. Specifically, not all—indeed hardly any—mathematical proofs are strict formally valid logical derivations. Of course, most of them can be restated in this manner, sometimes with comparatively little effort, but this is not something that mathematicians routinely do. To insist on such paraphrase is to misrepresent the nature of mathematical practice. Moreover, there is much that mathematicians do besides proving results, central as that activity may be. Most of this work may still be understood, however, as a species of argument.

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Deductive logic is concerned with validity and proof. It has been a formidably successful research programme within that context. However, this set of problems does not exhaust the subject matter of logic. Traditionally, logic has been understood as the study of argument. Deductive validity is one tool for the appraisal of argument. Other tools exist, including tools which permit finer-grained distinctions amongst the arguments classified as deductively invalid. In recent decades this topic has undergone something of a revival, and has acquired an increasingly thorough intellectual basis. It has also attracted a growing interest from other disciplines, notably artificial intelligence (see [7] especially). Landmarks include:


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Philosophy of mathematics needs an account of argument, and argumentation theory has one. This intersection has the potential to be hugely fruitful, but, with some important exceptions, remains largely unexplored. The exceptions are of two main types:

3.1. Some philosophers of mathematics have shown a considerable sensitivity to the structure of argument. Perhaps the most notable of these is Imre Lakatos [1], whose work was in turn influenced by that of George Polya [2, 3]. Work in this tradition does not typically cite argumentation research, although it often addresses closely related questions.
3.2. More recently, some authors have sought to integrate the two fields explicitly. These include researchers from an impressively diverse range of disciplines: not only philosophy of mathematics and argumentation theory, but also psychology, education and computer science.

There are numerous avenues for future research. Some possibilities include:

- Integration of existing work from different disciplines;
- Innovations in argumentation theory inspired by mathematical argument;
- Case studies of mathematical argument, informed by argumentation theory;
- Comparison of different techniques of reasoning employed within and across mathematical subdisciplines;
- Historical and/or intercultural case studies exhibiting the development of argumentational techniques in mathematics;
- Studies exhibiting the relation of techniques of argument in mathematics with similar techniques in wider use.