COURSE DESCRIPTION

The course presents partial differential equations starting from their physical origin and motivation. In particular it deals with the classical equations of mathematical physics, namely the wave equation, Laplace’s equation and heat equation. The structure of second order elliptic, parabolic and hyperbolic PDE’s is analyzed and the differencing mathematical features are investigated. A complete theory of nonlinear first order partial differential equations arising in continuum mechanics as conservation laws will be explained. Although the layout is theoretical, reference is continuously made to the underlying physics and the theory will proceed ”by examples” and ”physical models”. Even the standard classical topics in the theory of partial differential equations will be presented with particular emphasis on various modern approaches. Throughout the course the concept of nonlinearity and nonlinear analogues of the linear theory will be emphasized. Course will end with some important modern topics concerning second order nonlinear partial differential equations. A sizable practical part of the course is devoted to solving explicitly various physical problems either by separation of variables or by representation methods by means of fundamental solutions.
TEXTBOOK

No textbook is required. The class notes should suffice. A variety of topics and problems will be taken from the texts described in references at the end of syllabus. The recommended textbook is [1], while the practical problembook is [3] - both may be ordered via university bookshop.

GRADING POLICY

There will be three midterm exams and a final exam. Each midterm will be administered on the dates below, in the same classroom and at the same time as the scheduled lecture. The midterms will focus mainly on the material covered in the previous 4 weeks (or so). They consist of questions, theoretical or practical of the same type as those covered in class.

The final will be take home exam and it will be comprehensive. It is due on or before December 12 Wednesday. You will be given at least a week to do it. It will consist of two parts. The first will pose standard questions, theoretical or practical, of the same type as those covered during the semester. The second will have a few non-standard questions that are closely related to presented material. You are encouraged to consult texts, notes, books in the library, inquire with professors or other students about them.

Each of exam (midterm and final) will be graded in 100’s (i.e., the maximum score is 100). Your final score will be arithmetic average of your best 3 out of 4 scores. Your final grade will be determined by curving all final scores.

**Exam 1** Monday, September 17

**Exam 2** Monday, October 15

**Exam 3** Monday, November 5

**Final** Take Home Exam due Friday, December 7.
SYLLABUS

1 Some Mathematical Preliminaries

- Green’s Theorem
- Differential Operators and Adjoints
- The Cauchy-Schwarz-Young Inequality
- The Hölder Inequality
- Jensen’s Inequality
- The Inequality of the Geometric and Arithmetic Mean

2 Some Physical Models Leading to PDE’s

- The Continuity Equation
- The Heat Equation and the Laplace Equation
- A Model for the Vibrating String
- Small Vibrations of a Membrane
- Transmission of Sound Waves
- Fluid Dynamics and PDE’s
- The Navier-Stokes System
- The Euler Equation

3 The Laplace’s Equation

- Fundamental Solution
- Mean-value formulas
- Properties of Harmonic Functions
- Green’s Function
- Energy Methods
4 The Heat Equation

- Fundamental Solution
- Mean-value formula
- Properties of solutions
- Energy Methods

5 The Wave Equation

- Solution by Spherical Means
- Kirchhoff’s and Poisson’s Formulas
- Nonhomogeneous Problem
- Energy Methods

6 Nonlinear First-Order PDE

- Complete Integrals, Envelopes
- Characteristics
- Introduction to Conservation Laws

7 Classification of Second Order PDEs and Cauchy-Kovalevskaya Theorem

8 Linear Elliptic, Parabolic and Hyperbolic Equations

- Weak Solutions
- Energy Estimates
- Maximum Principles

9 Some Topics on Nonlinear PDEs
References


