SOBOLEV SPACES

MTH 6100, Summer 2011, TR 6:30 - 7:45 pm

Frederick Crawford Building 220

Professor Ugur G. Abdulla

Office Hours: S311, TR 5:30-6:30 pm and by appointment

COURSE DESCRIPTION

Partial differential equations (PDEs) are central to mathematics, whether pure or applied. They arise in mathematical models of real world problems, where dependent variables vary continuously as functions of several independent variables, usually space and time. Supported with the power of modern software tailored to suitable discretised approximations of the equations, applicability of the theory of PDEs penetrates all areas of modern science and technology and it is continuing to grow day by day.

In a majority of applications of both linear and nonlinear PDEs, non-smooth solutions are relevant. These are solutions which don't satisfy the PDE in a classical sense, but in a weak sense. Physically relevant weak solutions are the only solutions in many circunstances, and understanding them is not a matter of choice, but inevitability. Usually they do not belong to the classical spaces of continuously differentiable functions, rather to some other kinds of spaces containing less smooth functions. The course is an introduction to a study of properties of certain Banach spaces of weakly differentiable functions of several real variables which arise in connection with numerous problems in the theory and applications of partial differential equations. These spaces are associated with the name of the Soviet mathematician S.L.Sobolev, in view of his groundbreaking contributions to their development in the late 1930s.

The familiarity of the students with all the required prerequisite material from theory of functions and functionl analysis is not assumed and the topics on Lebesgue measure and measurable functions on $I\!R^N$, Lebesgue integration, L_p spaces, generalized (weak) derivatives and their main properties will be presented with proofs within the first half of the course. The second half of the course introduces the Sobolev spaces of weakly diffrentiable functions, and presents detailed analysis of approximation and extension theorems, and most importantly traces of Sobolev functions, Sobolev inequalities and compactness theorems. The course will end with some important application of Sobolev spaces method in analysis of partial differential equations.

TEXTBOOK

No textbook is required. The class notes should suffice. A variety of textbooks and monographs described in references can be consulted.

GRADING POLICY

Homework will be assigned periodically. Your performance will contribute to 20% of your final grade.

There will be one midterm and a final exam. Midterm exam will be administered on the date below, in the same classroom and at the same time as the scheduled lecture. The midterm will focus manly on the material covered in the previous 4 weeks (or so). It consists of questions, theoretical or practical of the same type as those covered in class.

The final will be take home exam and it will be comprehensive. It is due on or before July 7 Thursday. You will be given a week to do it.

Total score of 60 will be available from homework. Midterm Exam will be graded in 90's (30% of your grade), while final exam in 150's (50% of your grade). Hence, the maximum score is 300. Final grade will be determined by curving all final scores.

Midterm Exam Thursday, June 9

Final due to July 7.

SYLLABUS

Lebesgue's Measure and Measurable Sets in $\mathbb{R}^N$

Measurable Functions in $\mathbb{R}^N$

Lebesgue Integral and Integrable Functions

Fatou's Lemma and Lebesgue's Theorem

L_p Spaces and their main Properties

Generalized Derivatives and Weakly Differentiable Functions

Sobolev Spaces H^k and their main Properties

Approximation and Extension Results

Traces of Sobolev Functions

Compactness Theorems

Sobolev Type Inequalities and Applications

References

[1] L.C.Evans, Partial Differential Equations, Amer. Math. Soc., 2002.

[2] E.DiBenedetto, Partial Differential Equations, Birkhaser, 2010.

[3] R.A.Adams, Sobolev Spaces, Academic Press, 1975.

[4] V.Maz'ya, Sobolev Spaces, Springer Verlag, 1985.

[5] V.P.Mikhailov, Partial Differential Equations, Mir, 1978.

[6] J.S.Rosenthal, A First Look at Rigorous Probability Theory, World Scientific, 2006.

[7] P.Billingsley, Probability and Measure, John Wiley and Sons, 1995.

[8] K.Yosida, Functional Analysis, Springer-Verlag, 1995.

[9] I.P.Natanson, Theory of Functions of a Real Variable, Vol. I-II, New York, 1961.