PARTIAL DIFFERENTIAL EQUATIONS

MTH 5230, Spring 2013, MW 6:30 - 7:45 pm

Quadrangle Classrooms 113

Professor Ugur G. Abdulla

Office Hours: S319, MW 10:30-11:30 am and by appointment

COURSE DESCRIPTION

Partial differential equations (PDEs) are central to mathematics, whether pure or applied. They arise in mathematical models of real world problems, where dependent variables vary continuously as functions of several independent variables, usually space and time. Supported with the power of modern software tailored to suitable discretised approximations of the equations, applicability of the theory of PDEs penetrates all areas of modern science and technology and it is continuing to grow day by day. The course presents partial differential equations starting from their physical origin and motivation. In particular, it deals with the classical equations of mathematical physics, namely the wave equation, Laplace's equation and the heat equation, as well as first order partial differential equations arising in continuum mechanics as conservation laws. The course exposes the basic ideas critical to the study of PDEs – separation of variables, integral transforms, characteristics, Fourier series and related topics, Green functions. Reference is continuously made to underlying physics. Theory will only proceed "by examples" and "physical models". A sizable practical part of the course is devoted to solving explicitly various physical problems by using these methods. Students will be assigned an individual research projects concerning important applications of partial

differential equations in different scientific fields and engineering. The course will end with some important modern topic concerning second order nonlinear PDEs, which will prelude the modern theory of nonlinear partial differential equations.

TEXTBOOK

No textbook is required. The class notes should suffice. A variety of topics and problems will be taken from the texts described in references at the end of syllabus. The recommended textbooks are [1,2], while the practical problembook is [6]

GRADING POLICY

Homework will be assigned periodically. Your performance will contribute to 10% of your final grade.

There will be two midterm exams, assigned project presentation and a final exam. Midterm exam will be administered on the date below, in the same classroom and at the same time as the scheduled lecture. The midterm will focus manly on the material covered in the previous 5 weeks (or so). It consists of questions, theoretical or practical of the same type as those covered in class.

Within first two weeks you will be assigned an individual research project on application of partial differential equations in different scientific fields or engineering according to your interests. Final project presentation will be during class time on the dates below.

The final will be take home exam and it will be comprehensive. It is due on or before April 28 Monday. It will consist of two parts. The first will pose standard questions, theoretical or practical, of the same type as those covered during the semester. The second will have a few non-standard questions that are closely related to presented material. You are encouraged to consult texts, notes, books in the library, inquire with professors or other students about them.

Total score of 30 will be available from homework. Each of midterm exams and project presentation will be graded in 60's, and final exam in 90's (i.e., the maximum score is 300).

Final grade will be determined by curving all final scores.

1st Midterm Exam Monday, Feb 11

2nd Midterm Exam Monday, March 25

Project Presentation Monday, April 22 or Wednesday, April 24

Final Exam due to Monday, April 28.

SYLLABUS

Some Physical Models Leading to PDE's

• The Continuity Equation

• The Heat Equation and the Laplace Equation

• A Model for the Vibrating String

• Small Vibrations of a Membrane

• Transmission of Sound Waves

• Fluid Dynamics and PDE's

• The Navier-Stokes System

• The Euler Equation

Equations of the Hyperbolic Type

• Wave equation

• Initial-boundary value problems

• D'Alembert's formula

• Method of separation of variables

• Characteristics

Equations of the Parabolic Type

- Physical problems leading to heat/diffusion equation

- Formulation of boundary-value problems

- Maximum principle, uniqueness.

- Green's function

- Integral representation formula

- Free boundary problems

Equations of Elliptic Type

- Physical problems leading to Laplace's equation

- Potential flow in a fluid; potential of a stationary current and an electrostatic field

- General properties of harmonic functions

- Dirichlet and Neumann problems

Introduction to Nonlinear PDEs

- Physical problems leading to nonlinear diffusion equation

- Self-similar solutions

- Weak solutions and energy estimates

1 Topics for the individual research projects

- Free boundary model of laser ablation of biological tissues

- Modeling of short pulse laser ablation of skin tissues through nonlinear diffusion equation

- Identification of parameters in bioengineering

- Vibrations of strings of musical instruments

- Vibrations of rods

- Equations of gas dynamics and theory of shock waves

- The effect of radioactive decay on the temperature of the earth's crust

- The problem of freezing

- The Einstein-Kolmogorov equation and Brownian motion

- Problems of electrostatics

- Problem of electrical prospecting

- Diffusion of a cloud

- The demagnetization of a cylinder by a coil

- Application of the method of conforml mapping to hydrodynamics.

- Identification of parameters in biology: how fast potassium ions diffuse in an exoplasm solution or transmission of nerve impulses along axons.

- Monte Carlo solution of diffusion type equations: *tour du wino* game.

References

[1] L.C.Evans, Partial Differential Equations, Amer. Math. Soc.,Second Edition, 2010. ISBN10:0821849743. ISBN13:9780821849743.

[2] A.N. Tikhonov and A.A. Samarskii, Equations of Mathematical Physics, Dover, 2011. ISBN10:0-486-66422-8. ISBN13:978-0-486-66422-4.

[3] S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Dover, 1993. ISBN 0-486-67620-X.

[4] M. P. Coleman, An Introduction to Partial Differential Equations with MATLAB, Chapman & Hall/CRC, 2004. ISBN 1-58488-373-1.

[5] E.DiBenedetto, Partial Differential Equations, Birkhäuser, Boston, 1995.

[6] B.M.Budak, A.Samarskii and A.N.Tikhonov, A Collection of Problems in Mathematical Physics, Dover, 1964.

[7] R.Courant and D.Hilbert, Methods of Mathematical Physics, Vols. I and II, Interscience, New York, 1953, 1962.

[8] F.John, Partial Differential Equations, Springer-Verlag, New York, 1986.

[9] S.L.Sobolev, Partial Differential Equations of Mathematical Physics, Dover, 1989.

[10] V.P.Mikhailov, Partial Differential Equations, Mir, 1978.

[11] D.Gilbarg and N.Trudinger, Elliptic Partial Differential Equations of Second Order, Springer-Verlag, 1983.

[12] A.Friedman, Partial Differential Equations, Holt, Rinehart, Winston, 1969.

[13] U.G.Abdulla, Well-posedness of the Dirichlet problem for the nonlinear diffusion equation in non-smooth domains, *Trans. Amer. Math. Soc.*, 357, 1 (2005), 247-265.