Introduction to PDEs & Applications

MTH 3210, Summer 2013, TR 09:30 - 12:05 am

Fredrick C. Crawford Building 403

Professor Dr. Ugur G. Abdulla

Office Hours: Crawford-S319, TR 12:00-1:00pm and by appointment

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COURSE DESCRIPTION

Partial differential equations (PDEs) are central to mathematics, whether pure or applied. They arise in mathematical models of real world problems, where dependent variables vary continuously as functions of several independent variables, usually space and time. Supported with the power of modern software tailored to suitable discretised approximations of the equations, applicability of the theory of PDEs penetrates all areas of modern science and technology and it is continuing to grow day by day. The course presents partial differential equations starting from their physical origin and motivation. In particular, it deals with the classical equations of mathematical physics, namely the wave equation, Laplace's equation and the heat equation, as well as first order partial differential equations arising in continuum mechanics as conservation laws. The course exposes the basic ideas critical to the study of PDEs – separation of variables, integral transforms, special functions of the mathematical physics, characteristics and most importantly the Fourier series and related topics. A sizable practical part of the course is devoted to solving explicitly various physical problems by using these methods. Reference is continuously made to underlying physics. MATLAB software will be used for some important model problems and the Computing Lab of the Department of Mathematical Sciences will be available for students during some classes.

TEXTBOOK

The required textbook is

M. P. Coleman, An Introduction to Partial Differential Equations with MATLAB, Chapman & Hall/CRC, 2004. ISBN 1-58488-373-1.

Other recommended textbook is

S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Dover, 1993. ISBN 0-486-67620-X.

GRADING POLICY

Homework will be assigned periodically. Your performance will contribute to 10% of your final grade.

3 Quizzes will be given on the indicated dates. Quiz problems will be based on the homework problems. Your grade on those quizzes will contribute to 10% of your final grade.

There will be two midterm exams and a final exam. Each exam will be administered on the dates below, in the same classroom and at the same time as the scheduled lecture. The midterms will focus mainly on the material covered within the previous 2-3 weeks. They consist of practical problems of the same type as those covered in quizzes and homework. Your performance in each midterm exam will contribute to 25% of your final grade.

The two hour final exam is comprehensive. Your performance in final exam will contribute to 30% of your final grade.

Total score of 30 will be available from homework and another 30 from 3 quizzes; each midterm exam will be graded in 75's and final will be graded in 90's (i.e., the maximum score is 300). Your final grade will be determined by curving all final scores.

Exam 1: Thursday, May 30

Exam 2: Tuesday, June 18

Final Exam: Tuesday, July 2, 10am-12, Crawford Building 403.

SYLLABUS

1 Introduction

- What are Partial Differential Equations
- Initial and Boundary Conditions
- Linear PDEs-The Principle of Superposition
- Separation of Variables for Linear, Homogeneous PDEs
- Eigenvalue Problems

2 Major Three PDEs and Underlying Physics

- Second-Order, Linear, Homogeneous PDEs
- The Heat Equation and Diffusion
- The Wave Equation and the Vibrating String
- Transmission of Sound Waves
- Initial and Boundary Conditions for the Heat and Wave Equations
- Laplace's Equation–The Potential Equation

- Using Separation of Variables to Solve the Major Three PDEs

3 Solving the Major Three PDEs via Fourier Series

- The Fourier Series
- Completeness
- Homogeneous Heat Equation for a Finite Rod
- Homogeneous Wave Equation for a Finite String
- Homogeneous Laplace's Equation on a Rectangular Domain
- Nonhomogeneous Problems

4 Integral Transforms

- The Laplace Transform for PDEs
- Fourier Sine and Cosine Transforms
- The Fourier Transform
- The Heat Equation in Unbounded Regions
- Distributions, the Delta Function and Generalized Fourier Transforms

5 PDEs in Higher Dimensions and Special Functions of Mathematical Physics

- The Heat and Wave Equations on a Rectangle: Multiple Fourier Series
- Laplace's Equation in Polar Coordinates: Poisson Integral Formula
- The Wave and Heat Equations in Polar Coordinates. Bessel Functions
- Dirichlet Problem on a Ball, Cauchy-Euler Equation, Legendre Polynomials
- Diffusion of Heat in a Ball, Spherical Bessel's Equation, Harmonics.

Detailed Lecture Content & Homeworks

• **Lecture 1**–*May 14:* What are PDEs? PDEs we can already solve. Physical derivation of the Fourier Heat/Diffusion equation (1.1,1.2, partly 2.2)

• **Lecture 2**–*May 14:* Formulation of the main IBVPs for the heat equation. Dirichlet, Neumann and Robin boundary conditions(bc) and their physical meaning. Well-posedness of IBVPs. Linear PDEs. Homogeneous PDEs (1.3,1.4)

Homework 1. Exercise 1.1: 1,2e,3b,4b,5d,6c,6d; Ex. 1.2: 3,8,13,17,19; **due to May 21**

• **Lecture 3.**–*May 16:* The Principle of Superposition. Separation of Variables for Linear, Homogeneous PDEs. General solution of nonhomogeneous PDEs. Examples. (1.5,1.6)

Homework 2. Exercise 1.4: 3,4,6,9a,11; Exercise 1.5: 1,4,6,10; **due to May 23**

Homework 3. Exercise 1.6: Pairs 3 & 24, 5 & 26, 10, 16, 32; **due to May 23**

• **Lecture 4**–*May 16:* Eigenvalue problems. Example of 1D heat eq. on a finite rod with zero Dirichlet bc. Examples. (1.7)

Homework 4. Exercise 1.7: 3,6,15a,15c (including MATLAB); **due to May 23**

• **Lecture 5**–*May 21:* Physics of Wave equation: vibrating string, longitudinal vibrations, torsional vibrations. IBVPs for the wave equation. Dirichlet, Neumann and Robin boundary conditions and their physical meaning. Separation of variables, first BVP for the homogeneous wave equation, eigenvalue problems. (2.3,2.4,2.6)

• **Lecture 6**–*May 21:* Physics of Laplace equation: equation for the electrostatic potential, equation for the velocity potential of the incompressible fluid, stationary heat equation. BVPs for the Laplace equation: Dirichlet, Neumann and Robin BVPs. Separation of variables. (2.5,2.6)

Homework 5. Exercise 2.2: 10; Ex 2.4: 5,7; Ex 2.5: 3,5,6; Ex 2.6: 3,8,11; **due to May 23**

• **Lecture 7**–*May 23:* Fourier Series. Properties of the trigonometric family: periodicity, symmetry, orthogonality. Derivation of the Fourier coefficients formula. (3.1,3.2,3.3)

• **Lecture 8**–*May 23:* Piecewise continuous and piecewise smooth functions. Fourier convergence theorem. Fourier sine and cosine series. Completeness. (3.4,3.6)

• **Quiz 1**–*May 23:* Quiz problems are based on Homework 1-5.

Homework 6. Ex 3.2: 1,3,9; Ex 3.3: 1,3,7; Ex 3.4: 1,3,5; Ex 3.6: 1,3,5; **due to May 28**

• **Lectures 9 & 10**–*May 28* Practice test problems are solved.

• **Test 1**–*May 30*

• **MATLAB Session I**–*May 30:* MATLAB session will take place in Computing Lab. GSA runs the session and teaches students how to solve computational PDE problems using MATLAB.

• **Lecture 11**–*June 4:* Non-homogeneous problems (4.4).

Homework 7. Exercise 4.4: 1b,1d,4,8,16; **due to June 11**

• **Lecture 12**–*June 4:* The Laplace Transform for PDEs. Convolution Theorem (6.1)

• **Lecture 13**–*June 6:* The Fundamental Solution of the heat equation. Laplace Transforms of the fundamental solution, error function and the complementary error function. Solving BVPs on semi-infinite axis for the heat equation by using Laplace transform. Solving BVPs for the transport equation by using Lasplace transform. (6.1)

Homework 8. Exercise 6.1: 1b,2b,3b; **due to June 11**

• **Lecture 14**–*June 6:* Fourier Sine and Cosine transforms. (6.2)

Homework 9. Exercise 6.2: 2,4a,4b,5,7; **due to June 11**

• **Lecture 15**–*June 11:* The Fourier Transform. The main Theorem about the complex Fourier integral representation. Transforms of derivatives. Convolution. Translation.(6.3)

• **Quiz 2**–*June 11:* Quiz problems are based on Homework 6-9.

Homework 10. Exercise 6.3: 1c,1e,3,9,11a,11d; **due to June 18**

• **Lecture 16**–*June 13:* Solving PDE problems via Fourier transformation. Cauchy Problem for the Heat/Diffusion equation. Method of Images for solving heat equation on a semiaxis with Dirichlet and Neumann boundary conditions. Diffusion-Convection equation. Euler-Bernoulli beam equation. (6.4)

Homework 11. Exercise 6.4: 1,3,4a,4b,4c,4d,5,6b,7,8; **due to June 18**

• **Lecture 17**–*June 13:* D'Alembert's formula for the wave equation; Laplace's equation on the half-plane: Poisson's integral formula for the upper half-plane, Poisson kernel.

• **Test 2**–*June 18*

• **MATLAB Session II**–*June 18:* MATLAB session will take place in Computing Lab. GSA runs the session and teaches students how to solve computational PDE problems using MATLAB.

• **Lecture 18**–*June 20:* The Heat and Wave Equations on a rectangle. Multiple Fourier series. (9.2)

Homework 12. Exercise 9.2: 1,3a,4a,5c,5d,6a,6d,9a,9d; **due to June 25**

• **Lecture 19**–*June 20:* Laplace's equation in polar coordinates. Dirichlet Problem on a disk. Poisson Integral formula. Mean Value formula. Geometric and physical meaning of the Poisson Integral. (9.3)

Homework 13. Exercise 9.3: 1,2,4,6,8a,8b,16a,16b; **due to June 25**

• **Lecture 20**–*June 25:* The Gamma Function. Method of Frobenius. Solving Bessel ODE via Frobenius method. Bessel functions of the first kind. Bessel function of the second kind (or Weber function) (7.4,7.5). Note: modified Bessel ODE goes to homework.

• **Lecture 21**–*June 25:* The Wave and Heat Equations in polar coordinates. Solving Wave and Heat equations on a cylinder with circular cross-section. Fourier-Bessel series.(9.4)

• **Quiz 3**–*June 25:* Quiz problems are based on Homework 10-13.

Homework 14. Ex 7.3: 8a,8b; Ex 7.5: 9a,9b,9c,9d; Ex 9.4: 1b,2a,4a,6a; **due to July 2**

• **Lecture 22**–*June 27:* Spherical coordinates. Laplace equation in spherical coordinates. General Dirichlet problem on a ball. Legendre's ODE. Solving the eigenvalue problem for the Legendre's equation via power series method. Legendre's polynomials. Solving the Dirichlet problem on a ball. Fourier-Legendre series. (7.2,9.5). Note: associated

Legendre's equation goes to homework.

• **Lecture 23**–*June 27:* Review for Final.

Homework 15. Ex 7.2: 2,5a,5b,5c; Ex 9.5: 1,3b,4a,4b,8; **due to July 2**

• **Final Exam**–*July 2*