Introduction to PDEs & Applications

MTH 3210–01, Spring 2013, TR 08:00 - 09:15 am

George M. Skurla Hall 106

Professor Dr. Ugur G. Abdulla

Office Hours: Crawford-S319, TR 09:30-10:30am and by appointment

GSA: Ashley Ferguson

GSA's office hours: S310 or S330, Mo–Fr 9:30-11am and by appointment

COURSE DESCRIPTION

Partial differential equations (PDEs) are central to mathematics, whether pure or applied. They arise in mathematical models of real world problems, where dependent variables vary continuously as functions of several independent variables, usually space and time. Supported with the power of modern software tailored to suitable discretised approximations of the equations, applicability of the theory of PDEs penetrates all areas of modern science and technology and it is continuing to grow day by day. The course presents partial differential equations starting from their physical origin and motivation. In particular, it deals with the classical equations of mathematical physics, namely the wave equation, Laplace's equation and the heat equation, as well as first order partial differential equations arising in continuum mechanics as conservation laws. The course exposes the basic ideas critical to the study of PDEs – separation of variables,

integral transforms, special functions of the mathematical physics, characteristics and most importantly the Fourier series and related topics. A sizable practical part of the course is devoted to solving explicitly various physical problems by using these methods. Reference is continuously made to underlying physics. MATLAB software will be used for some important model problems and the Computing Lab of the Department of Mathematical Sciences will be available for students during some classes.

TEXTBOOK

The required textbook is

M. P. Coleman, An Introduction to Partial Differential Equations with MAT-LAB, Chapman & Hall/CRC, 2004. ISBN 1-58488-373-1.

Other recommended textbook is

S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Dover, 1993. ISBN 0-486-67620-X.

GRADING POLICY

Homework will be assigned periodically. Your performance will contribute to 20% of your final grade.

There will be two midterm exams and a final exam. Each midterm will be administered on the dates below, in the same classroom and at the same time as the scheduled lecture. The midterms will focus mainly on the material covered within the previous 4-5 weeks. They consist of practical problems of the same type as those covered in quizzes and homework. Your performance in each midterm exam will contribute to 25% of your final grade.

The two hour final exam is comprehensive. It will be administered on the date below, in the same classroom. Material covered after the second midterm test will form 50%

of the final exam. Your performance in final exam will contribute to 30% of your final grade.

Total score of 60 will be available from homework and quizzes; each midterm exam will be graded in 75's and final will be graded in 90's (i.e., the maximum score is 300). Your final grade will be determined by curving all final scores.

Exam 1: Thursday, February 14

Exam 2: Tuesday, March 26

Final Exam: Friday, May 3, 1-3m, Skurla Hall 106.

SYLLABUS

1 Introduction

- What are Partial Differential Equations

- Initial and Boundary Conditions

- Linear PDEs-The Principle of Superposition

- Separation of Variables for Linear, Homogeneous PDEs

- Eigenvalue Problems

2 Major Three PDEs and Underlying Physics

- Second-Order, Linear, Homogeneous PDEs

- The Heat Equation and Diffusion

- The Wave Equation and the Vibrating String

- Transmission of Sound Waves

- Initial and Boundary Conditions for the Heat and Wave Equations

- Laplace's Equation–The Potential Equation

- Using Separation of Variables to Solve the Major Three PDEs

3   Solving the Major Three PDEs via Fourier Series

- The Fourier Series

- Completeness

- Homogeneous Heat Equation for a Finite Rod

- Homogeneous Wave Equation for a Finite String

- Homogeneous Laplace's Equation on a Rectangular Domain

- Nonhomogeneous Problems

4   Integral Transforms

- The Laplace Transform for PDEs

- Fourier Sine and Cosine Transforms

- The Fourier Transform

- The Heat Equation in Unbounded Regions

- Distributions, the Delta Function and Generalized Fourier Transforms

5   PDEs in Higher Dimensions and Special Functions of Mathematical Physics

- The Heat and Wave Equations on a Rectangle: Multiple Fourier Series

- Laplace's Equation in Polar Coordinates: Poisson Integral Formula

- The Wave and Heat Equations in Polar Coordinates. Bessel Functions

- Dirichlet Problem on a Ball, Cauchy-Euler Equation, Legendre Polynomials

- Diffusion of Heat in a Ball, Spherical Bessel's Equation, Harmonics.

Detailed Lecture Content & Homeworks

• **Lecture 1**–*Jan 8:* What are PDEs? PDEs we can already solve. Physical derivation of the Fourier Heat/Diffusion equation (1.1,1.2, partly 2.2)

• **Lecture 2**–*Jan 10:* Formulation of the main IBVPs for the heat equation. Dirichlet, Neumann and Robin boundary conditions(bc) and their physical meaning. Well-posedness of IBVPs. Linear PDEs. Homogeneous PDEs (1.3,1.4)

Homework 1. Exercise 1.1: 1,2e,3b,4b,5d,6c,6d; Ex. 1.2: 3,8,13,17,19; **due to Jan 15**

• **Lecture 3.**–*Jan 15:* The Principle of Superposition. Separation of Variables for Linear, Homogeneous PDEs. General solution of nonhomogeneous PDEs. Examples. (1.5,1.6)

Homework 2. Exercise 1.4: 3,4,6,9a,11; Exercise 1.5: 1,4,6,10; **due to Jan 22**

Homework 3. Exercise 1.6: Pairs 3 & 24, 5 & 26, 10, 16, 32; **due to Jan 22**

• **Lecture 4**–*Jan 17:* Eigenvalue problems. Example of 1D heat eq. on a finite rod with zero Dirichlet bc. Examples. (1.7)

Homework 4. Exercise 1.7: 3,6,15a,15c (including MATLAB); **due to Jan 24**

• **Lecture 5**–*Jan 22:* Physics of Wave equation: vibrating string, longitudional vibrations, torsional vibrations. IBVPs for the wave equation. Dirichlet, Neumann and Robin boundary conditions and their physical meaning. Separation of variables, first BVP for the homogeneous wave equation, eigenvalue problems. (2.3,2.4,2.6)

• **Lecture 6**–*Jan 24:* Physics of Laplace equation: equation for the electrostatic potential, equation for the velocity potential of the incompressible fluid, stationary heat equation. BVPs for the Laplace equation: Dirichlet, Neumann and Robin BVPs. Separation of variables. (2.5,2.6)

Homework 5. Exercise 2.2: 10; Ex 2.4: 5,7; Ex 2.5: 3,5,6; Ex 2.6: 3,8,11; **due to Jan 31**

• **Lecture 7**–*Jan 29:* Fourier Series. Properties of the trigonometric family: periodicity, symmetry, orthogonality. Derivation of the Fourier coefficients formula. (3.1,3.2,3.3)

• **Lecture 8**–*Jan 31:* Piecewise continuous and piecewise smooth functions. Fourier

convergence theorem. Fourier sine and cosine series. Completeness. (3.4,3.6)

Homework 6. Ex 3.2: 1,3,9; Ex 3.3: 1,3,7; Ex 3.4: 1,3,5; Ex 3.6: 1,3,5; **due to Feb 7**

• **Lecture 9**–*Feb 5 and 7* is shared with MATLAB session in the Computing Lab. Half of the class attends the lecture, while the other half attends the MATLAB session. GSA runs the sessions and teaches students how to solve computational PDE problems using MATLAB. The purpose of the lecture is to solve a test oriented sample quiz. The quiz consists of three problems for heat, wave and Laplace's equation. Problems 1 & 2 of the Quiz are solved during class time.

• **Lecture 10**(*Feb 12*) Problem 3 of the sample Quiz is solved. More test oriented sample problems are solved.

• **Test 1**–*Feb 14*

• **Lecture 11**–*Feb 19:* Non-homogeneous problems (4.4).

Homework 7. Exercise 4.4: 1b,1d,4,8,16; **due to Feb 26**

• **Lecture 12**–*Feb 21:* The Laplace Transform for PDEs. Convolution Theorem (6.1)

• **Lecture 13**–*Feb 26:* The Fundamental Solution of the heat equation. Laplace Transforms of the fundamental solution, error function and the complementary error function. Solving BVPs on semi-infinite axis for the heat equation by using Laplace transform. Solving BVPs for the transport equation by using Lasplace transform. (6.1)

Homework 8. Exercise 6.1: 1b,2b,3b; **due to March 12**

• **Lecture 14**–*Feb 28:* Fourier Sine and Cosine transforms. (6.2)

Homework 9. Exercise 6.2: 2,4a,4b,5,7; **due to March 12**

• **Lecture 15**–*March 12:* The Fourier Transform. The main Theorem about the complex Fourier integral representation. Transforms of derivatives. Convolution. Translation.(6.3)

Homework 10. Exercise 6.3: 1c,1e,3,9,11a,11d; **due to March 19**

• **Lecture 16**–*March 14:* Solving PDE problems via Fourier transformation. Cauchy Problem for the Heat/Diffusion equation. Method of Images for solving heat equation

on a semiaxis with Dirichlet and Neumann boundary conditions. Diffusion-Convection equation. Euler-Bernoulli beam equation. (6.4)

Homework 11. Exercise 6.4: 1,3,4a,4b,4c,4d,5,6b,7,8; **due to March 21**

• **Lecture 17**(*March 19 and 21*) is shared with MATLAB session in the Computing Lab. Half of the class attends the lecture, while the other half attends the MATLAB session. GSA runs the sessions and teachs students how to solve computational PDE problems using MATLAB. The lecture covers two topics: Derivation of d'Alembert's formula for the wave equation via Fourier transformation. Solving Laplace's equation on the half-plane: Poisson's integral formula for the upper half-plane, Poisson kernel.

• **Test 2**–*March 26*

• **Lecture 18**–*March 28:* The Heat and Wave Equations on a rectangle. Multiple Fourier series. (9.2)

Homework 12. Exercise 9.2: 1,3a,4a,5c,5d,6a,6d,9a,9d; **due to April 4**

• **Lecture 19**–*April 2:* Laplace's equation in polar coordinates. Dirichlet Problem on a disk. Poisson Integral formula. Mean Value formula. Geometric and physical meaning of the Poisson Integral. (9.3)

Homework 13. Exercise 9.3: 1,2,4,6,8a,8b,16a,16b; **due to April 9**

• **Lecture 20**–*April 4:* The Gamma Function. Method of Frobenius. Solving Bessel ODE via Frobenius method. Bessel functions of the first kind. Bessel function of the second kind (or Weber function) (7.4,7.5). Note: modified Bessel ODE goes to homework.

• **Lecture 21**–*April 9:* The Wave and Heat Equations in polar coordinates. Solving Wave and Heat equations on a cylinder with circular cross-section. Fourier-Bessel series.(9.4)

Homework 14. Ex 7.3: 8a,8b; Ex 7.5: 9a,9b,9c,9d; Ex 9.4: 1b,2a,4a,6; **due to April 16**

• **Lecture 22**–*April 11:* Spherical coordinates. Laplace equation in spherical coordinates. General Dirichlet problem on a ball. Legendre's ODE. Solving the eigenvalue problem for the Legendre's equation via power series method. Legendre's polynomials. Solving the Dirichlet problem on a ball. Fourier-Legendre series. (7.2,9.5). Note:

associated Legendre's equation goes to homework.

Homework 15. Ex 7.2: 2,5a,5b,5c; Ex 9.5: 1,3a,3b,3c,4a,4b,8; **due to April 23**

- **Lecture 23**(*April 16 and 18*) is shared with MATLAB session in the Computing Lab. Half of the class attends the lecture, while the other half attends the MATLAB session. GSA runs the sessions and teachs students how to solve computational PDE problems using MATLAB. The lecture covers: Diffusion of Heat in a Ball. Solving 3D Heat/Diffusion equation in a ball. Associated Legendre's ODE. Spherical Bessel's equation. Spherical Bessel's functions of the first kind. Harmonics. (9.5). Note: solving wave equation in a ball goes to homework.

- **Lecture 24**–*April 23:* Review for Final.

- **Final Exam**–*May 3*