Personal Communication Systems (Lecture 4)

Last time: (two-ray model)

\[ \text{RSL (dBu)} = \text{Rx (dBu)} + 20 \log H_x + 20 \log H_y - 40 \log d \]  
(1)

or

\[ \text{Pt (dB)} = 40 \log d - 20 \log H_x - 20 \log H_y \]  
(2)

Equation (2) does not take into account two very important aspects of propagation:

1. Non-field effects (signal attenuates much faster than 40dB/dec)
2. Effects of trees, obstructions (large objects existing along the radio path)

The easiest way to take the loss into account is through the Log distance path loss model. According to this model, the path loss equation is in the form

\[ \text{PL}(d) (dB) = \text{PL}(d_0) + 10 \log (d/d_0) + X_5 \]

where

- \( d_0 \) - reference distance
- \( d \) - distance
- \( \text{PL}(d_0) \) - average path loss at the reference distance
- \( X \) - path loss exponent
- \( X_5 \) - random remaining of the path loss
- \( X_5 \) - random variable usually assumed as having log-normal distribution
\[ \text{RSS}(d) = \text{Rx}[\text{dBm}] - \text{PL}(d) [\text{dB}] \]

- near field effects
- speed is proportional to \( C \)
- slope of the line \( n \) \( \text{dB/dB} \)

Observations:

- after reference distance \( (d_0) \), path loss has a linear decay as a function of distance
- for various \( X \) add path loss character to the path loss modeling - we give up on trying to predict the path loss analytically

Some typical values for path loss exponent:

- free space \( n = 2 \)
- flat Earth \( n = 4 \)
- cellular indoor environment \( n = 3.5 \)
- forests \( n = 6 \)
- in building (los) \( 40 - 100 \)
- rural environment (los) \( 16 - 25 \)
Quantity \[ m \] is referred to as the shape.

So, in a typical cell environment, one would typically encounter path loss slopes on the order 20–50 dB/dec.

Properties of \( X_5 \):

\[ X_5 \sim \mathcal{N}(0, 5) \] is a normal distribution with 0 mean and standard deviation of 5.

\[ X_5 \sim \text{pdf}(x) = \frac{1}{\sqrt{2\pi} \cdot 5} \exp\left(-\frac{x^2}{2 \cdot 5^2}\right), \quad x \in (-\infty, +\infty) \]

\[ \mu = 0 \]

\[ \sigma^2 = 5 \]

Some typical values for \( 5 \):

<table>
<thead>
<tr>
<th>Environment</th>
<th>( 5 ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>5–7</td>
</tr>
<tr>
<td>Suburban</td>
<td>6–8</td>
</tr>
<tr>
<td>Urban</td>
<td>8–10</td>
</tr>
<tr>
<td>Dense urban</td>
<td>10–12</td>
</tr>
</tbody>
</table>
Random component $X_6$ is sometimes referred to as the log-normal fading or shadowing.

Example 1. Consider a cell site with an ERP $= 50$ dBm. Assume that the path loss follows a log-distance path loss model. The following numerical data are given

$$PL(d_w) = 109 \text{ dB}$$
$$d_w = 1 \text{ mile}$$
$$m = 10 \log = 38.4 \text{ dB/km}$$

a) Calculate the RSL at distances of $d = 2$ miles.
b) Calculate the probability that the signal is above the level calculated in (a).

a) $\begin{align*}
PL(d) &= PL(d_w) + 10 \log d + X_6 \\
PL(2) &= PL(d_w) + 10 \log 2 \\
&= 109 + 20.41 \log(2) = 127.32 \text{ dB}
\end{align*}$

RSL = ERP [dBm] - PL [dB] = $50 \text{ dBm} - 127.32 \text{ dB} = -77.32 \text{ dBm}$

b) $\begin{align*}
\text{RSL} = -77.32 \text{ dBm}
\end{align*}$

$\text{Mean} = 109 \text{ dB}$. The ordinates are symmetrical and have a mean deviation in log domain.
Therefore, at distance of 2 miles approximately half of measurements will be above -77.52 dBm and half of the measurements will be below -17.32 dBm.

Another way to visualize the same thing is to plot the contours of the same RSL.

* Note that the contours are not circles or hexagons.
* Cells do not have regular shape.

Example 2: The RSL predicted by log-distance model worldwide is -80 dBm. Assume that the log-normal shadowing component has standard deviation $\sigma = 7$ dB. Calculate the probability.

a) $RSL > -80$ dBm
b) $RSL < -80$ dBm
c) $RSL > -85$ dBm
d) $RSL > -75$ dBm

e) $RSL \sim \mathcal{N}(RSL,\sigma)$

$$P_{\text{h}}(RSL > RSL_{\text{h}}) = \int_{RSL_{\text{h}}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-RSL)^2}{2\sigma^2}\right) dx$$

In this case,
\[ P_{\Delta RSL > \delta} = \int_{\delta}^{\infty} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = 0.5 \]

b) \[ P_{\Delta RSL < -8} = 0.5 \]

c) \[ P_{\Delta RSL > -8} = \int_{-8}^{\infty} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \]

Algorithm

1. Substitution \[ t = \frac{x-\mu}{\sigma} = \frac{x-(-80)}{7} \]

\[ dt = \frac{dx}{\sigma} \quad \rightarrow \quad dx = 7 \, dt \]
\[ \Pr \{ \text{PSL} > -85 \text{dBm} \} = \int_{-85}^{-\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2}) \, dt \]

\[ = -0.7143 \int_{-85}^{-\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2}) \, dt = 0.7625 \]

*Review Normal Distribution table - posted on the Web*

1) \[ \Pr \{ \text{PSL} > -75 \text{ dBm} \} = 0.2875 \]