Elements of RF propagation.

Propagation of RF signals - very complicated phenomena that can be approximately handled in a limited number of cases.

Propagation in free space:
- Assumes clear/undistorted path between TX & RX
- Partly happens in terrestrial communication
- Two important examples of free space propagation
  - Satellite communication
  - UHF communication

\[ \text{Power density of the RF signal at distance } d \text{ from the TX:} \]
\[ \frac{1}{W/\text{m}^2} = \frac{1}{	ext{E} \times \text{A}} = \frac{1}{\text{E}^2} = \frac{P_r}{\lambda^2 \cdot 4 \pi d^2}, \text{ where } \eta = 120 \pi - 	ext{characteristic impedance of free space} \]
The RX antenna collects energy. The energy transmitted to the matched receiver load is proportional to the power density of the RF signal and the effective aperture of the receive antenna. That is

$$P_{Rx} = W \times Ac = \left[ \frac{G_{Rx} \cdot G_{x}}{4 \pi d^2} \right] \times \frac{G_{x} \cdot A^2}{4\pi}$$

$$= \frac{P_{Tx} \cdot G_{Rx} \cdot G_{x}}{(4\pi \lambda)^2} \quad (2)$$

where $\lambda$ = $\frac{c}{f}$, the wavelength of the radio waves.

$$P_{Rx} = \frac{G_{Rx} \cdot G_{x}}{(4\pi \lambda)^2} \quad (3)$$

Using (3), one can define the path loss as

$$PL = P_{Rx}/P_{Tx} = \frac{(4\pi \lambda)^2}{G_{Rx} \cdot G_{x}} \quad (4)$$

It is common to express the path loss in dB

$$PL[dB] = 10 \log \left( \frac{P_{Rx}}{P_{Tx}} \right) = 10 \log \left( \frac{(4\pi \lambda)^2}{G_{Rx} \cdot G_{x}} \right) - 10 \log G_{Rx} - 10 \log G_{x} \quad (5)$$

Free space loss (FSPL) = Loss due to the atmosphere

$$FSPL[dB] = 20 \log \left[ \frac{4\pi d}{\lambda} \right] = 20 \log \left( \frac{d}{\lambda} \right) \quad (6)$$

or

$$FSPL[dB] = 36.8 + 20 \log d + 20 \log \frac{4\pi d}{\lambda} \quad (7)$$

Fresnel region

$$FSPL[dB] = 82.4 + 20 \log d + 20 \log \frac{4\pi d}{\lambda} \quad (8)$$
Important points

1) Path loss increases 20 dB/dec with distance

Example: \[ PL = \text{initial} - 10 \times \text{dec} \]

\[ \begin{align*}
\text{PL [initial]} &= 100 \text{dB} \\
\text{PL [transmit]} &= 120 \text{dB} \quad \text{Small path loss results}
\end{align*} \]

\[ \begin{align*}
\text{PL [transmit]} &= 140 \text{dB} \\
\text{PL [receiving]} &= 160 \text{dB} \quad \text{large distance}
\end{align*} \]

2) Path loss increases 20 dB/dec with the increase of frequency

Example: Consider a wireless communication link

\[ \begin{align*}
\text{Gx} &= 20 \text{dB} \\
\text{Gy} &= 5 \text{dB} \\
\text{GSLawm} &= -80 \text{dB}
\end{align*} \]

Received signal level

\[ \begin{align*}
\text{Calculates the opening frequency:} & \quad f = 1900 \text{MHz} \\
& \quad f = 2.8 \text{GHz} \\
& \quad f = 6 \text{GHz}
\end{align*} \]

Effective radiated power:

\[ \text{ERP (dBm)} = \text{Prx (dBm)} + \text{Gx (dB)} \]

\[ \begin{align*}
\text{ERP (dBm)} &= 36 \text{dBm} + 20 \text{dB} = 56 \text{dBm} \\
\text{GSLawm (borne air)} &= -80 \text{dBm} - 5 \text{dB} = -85 \text{dBm}
\end{align*} \]

Minimum FSPL:

\[ \text{FSPL} = \text{ERP (dBm)} - \text{GSLawm (borne air)} = 55 \text{dBm} - (-85 \text{dBm}) = 140 \text{dB} \]

\[ \text{FSPL} = 366 + 20 \log \left( \text{f} \left( \text{MHz} \right) \right) + 20 \log \left( \text{d} \left( \text{km} \right) \right) \]
\[ f = 1939 \text{ MHz} \]

\[ 13 \text{ dB} = 20 \log(1939) + 20 \log(d \text{ miles}) \Rightarrow d = 61.8 \text{ miles} \]

\[ f = 2.4 \text{ GHz} \]

\[ 13 \text{ dB} = 20 \log(2400) + 20 \log(d \text{ miles}) \Rightarrow d = 48.95 \text{ miles} \]

\[ f = 6 \text{ GHz} \Rightarrow d = 19.38 \text{ miles} \]

Note: Maximum distance decreases with an increase in frequency.

Propagation in Flat Earth:

4. First order approximation of the spherical propagation
4. Earth is assumed
   - Flat (no Earth curvature)
   - Superconducting cords or a superconductor
   - Hard (no objects on the surface)

\[ \text{diag. wave} \]
A signal consists of two waves
- direct wave
- reflected wave

A reflected wave is phase shifted relative to direct wave due to
- reflection from earth (-0.5 knot shift)
- difference in propagation path.

\[
P_{ex} = \frac{P_{ex} \cdot G_{ex}}{(4\pi d)^2} \left[ 1 - \exp\left(-\frac{\lambda}{\lambda} \frac{2\pi d}{V} \right) \right]
\]

Direct wave - free space
loading due to superposition of
direct wave and reflected wave

The path difference (From the figure)

\[
d = \sqrt{(H_{tx} + H_{rx})^2 + d^2} - \sqrt{(H_{tx} - H_{rx})^2 + d^2}
\]

\[
d = \sqrt{1 + \left(\frac{H_{tx} + H_{rx}}{d}\right)^2} - \sqrt{1 + \left(\frac{H_{tx} - H_{rx}}{d}\right)^2}
\]

Assuming \( d \gg H_{tx} + H_{rx} \), and using \( \sqrt{1+x} \approx 1 + \frac{x}{2} \)

\[
d = \frac{1}{2d^2} \left[ 1 + \left(\frac{H_{tx} + H_{rx}}{d}\right)^2 \right] - \frac{1}{2d^2} \left[ 1 + \left(\frac{H_{tx} - H_{rx}}{d}\right)^2 \right]
\]

\[
d = \frac{\frac{H_{tx}^2 + 2H_{tx}H_{rx} + H_{rx}^2}{2d^2} - \frac{H_{tx}^2 + 2H_{tx}H_{rx} + H_{rx}^2}{2d^2}}{2d^2}
\]

\[
d = \frac{H_{rx}^2}{2d^2}
\]

Substituting (11) into (9) one obtains

\[
P_{ex} = \frac{P_{ex} \cdot G_{ex}}{(4\pi d)^2} \left[ 1 - \exp\left(-\frac{\lambda}{\lambda} \frac{2\pi d}{V} \right) \right]
\]

\[
\approx \frac{P_{ex} \cdot G_{ex}}{(4\pi d)^2} \left[ 1 - 1 \right] \frac{2H_{rx} H_{rx}}{d}
\]
\[
\text{where } \sigma(x) = 1 + x + 5(x) \text{ for small } x \text{ is used.}
\]

**Theorem:**

\[
P_{rx} = P_{tx} G_{rx} G_{tx} \left( \frac{H_{rx} H_{tx}}{\lambda} \right)^2 = P_{tx} \left( \frac{L_{rx} H_{rx}}{d_1^2} \right)^2 G_{rx} G_{tx}
\]

\[
P_L = P_{tx}/P_{rx} = \frac{d_1^2}{(H_{rx} H_{tx})^2} G_{rx} G_{tx}
\]

or in dB

\[
P_L [\text{dB}] = 10 \log \left( \frac{d_1^2}{(H_{rx} H_{tx})^2} G_{rx} G_{tx} \right)
\]

*important notes*

1. **Path loss increases 40 dB/dec as a function of distance.**

   **Example:**
   
   \[
P_L [\text{dB}] = 100 \text{ dB}
   \]
   
   \[
P_L [\text{dB}] = 140 \text{ dB}
   \]

2. **Path loss is a function of TX and RX heights.**

   **Example:** Consider an omni-directional collinear with the following numerical data: transmit power, \(P_{tx} = 100\), transmit antenna gain, \(G_{tx} = 10 \text{ dB}\), and TX height, \(H_{tx} = 100 \text{ ft}\). If the receive antenna gain is \(G_{rx} = 5 \text{ dB}\), calculate the cell radius.

   If the receive antenna height is \(54 \text{ ft}\), the receive antenna gain is \(0 \text{ dB}\) and transmit power is \(20 \text{ dBm}\), calculate the cell radius.
\[ PSL(\text{dBm}) = ERP(\text{dBm}) - PL(\text{dB}) \]

\[ ERP(\text{dBm}) = P_{\text{tx}}(\text{dBm}) + 20 \log_2 I = 10 \text{ dBm} + 10 \log_2 1 = 50 \text{ dBm} \]

\[ PL_{\text{max}}(\text{dB}) = ERP(\text{dBm}) - PSL_{\text{min}}(\text{dB}) = 50 \text{ dBm} - (-80 \text{ dBm}) = 130 \text{ dB} \]

\[ P_{\text{max}} = 1 \text{ W} \cdot 20 \log_2 d - 20 \log_2 H_T - 20 \log_2 H_R \]

\[ 130 \text{ dB} = 10 \log_2 d - \frac{20 \log_2 (\frac{100}{8})}{8} = 20 \log_2 (\frac{5}{\omega}) \left( \frac{\text{rad}}{\text{s}} \right) \left\{ \text{radians for units} \right\} \]

\[ d = 12.23 \text{ m} = 7.53 \text{ miles} \]