Interconnecting Systems

1) Parallel connection

![Parallel connection diagram]

\[ h_{\text{out}} = h_1[n] + h_2[n] \]

2) Series connection

![Series connection diagram]

\[ h_{\text{out}} = h_1[n] \times h_2[n] \]

Response of DT system to decaying exponential

\[ x[n] = 2^n \]

\[ y[n] = 2^n \times h[n] \]

\[ 2^n - \text{exponential, in general case } 2 = \alpha + j\beta = \rho e^{j\phi} \text{ - complex number} \]

\[ y[n] = 2^n \times h[n] = \sum_{m=-\infty}^{\infty} h[m] \cdot 2^{n-m} = 2^n \sum_{m=-\infty}^{\infty} h[m] \cdot 2^{-m} \]

\[ = 2^n \cdot H(2) \quad H(z) \text{ - referred to as the transfer function} \]
Another way to define $H(z)$ is:

$$H(z) = \frac{\text{output signal}}{\text{input signal}} \quad \text{given initial conditions, } x[n] = 2^n - \text{input}$$

Consider:

$$O(z) \cdot y[n] = P(z) \cdot x[n]$$

Let $x[n] = 2^n$; then:

$$O(z) \cdot y[n] = O(z) \cdot \sum_{j=0}^{\infty} h_j \cdot 2^n = H(z) \cdot O(z) \cdot 2^n$$

$$P(z) \cdot x[n] = P(z) \cdot 2^n$$

Therefore:

$$H(z) = \frac{P(z)}{O(z)}$$

- Obtaining of the transfer function directly from the difference equation of the system.

**Total response of LTI system**

\[ y[n] = y_2[n] + y_3[n] = \sum_{j=1}^{\infty} h_j \cdot 2^n + x[n] \cdot h[n] \]

- Natural modes.

- Convolution of the input and impulse response of the system.
Example. Consider system

\[ y[n+2] - 0.6 y[n+1] - 0.16 y[n] = 5x[n+2] \quad (x) \]

Initial conditions: \( y[-1] = 0, \ y[-2] = 2 \leq 4 \)

Input: \( x[n] = 4^{-n} u[n] \)

Step 1. Find input response

\[ \omega(n) = y^2 - 0.6 y - 0.16 \]

\[ y_1 = \frac{0.6 \pm \sqrt{0.36 + 0.4}}{2} = \frac{0.6 \pm \sqrt{0.4}}{2} \]

\[ y_1 = -0.2, \ y_2 = 0.8 \]

\[ y_2[n] = c_1 (0.2)^n + c_2 (0.8)^n \]

Step 2. Determine impulse response.

\[ (E^2 - 0.6 E - 0.16) y[n] = 5 E^2 x[n] \]

\[ h[n] = b[n] \delta[n] + y_c[n] u[n] \]

\[ h[n] = y_c[n] = k_1 (0.2)^n + k_2 (0.8)^n \]

Since \( b[n] = 0 \) \( \#\)

To determine \( k_1 \) and \( k_2 \), assume \( x[n] = \delta[n] \), from (x)

one has: \( y[n+2] = 0.6 y[n+1] + 0.16 y[n] + 5 x[n+2] \) or

\[ y[n] = 0.6 y[n-1] + 0.16 y[n-2] + 5 x[n] \]

Thus, we have

\[ h[n] = 0.6 h[n-1] + 0.16 h[n-2] + 5 \delta[n] \]
(79)

\[ u(t) = 5. \]
\[ \mu[1] = 0.6 \mu[0] + 0.16 \mu[-1] + 5 \mu[1] = 0.6 \times 5 = 3. \]

From (x)
\[ h(0) = k_1 + k_2 = 5 \]
\[ k_1 = 1 \]
\[ k_2 = 4 \]

Therefore
\[ h[n] = \left\{ \begin{array}{l}
  (0.2)^n + 4, 0.8^n \end{array} \right\} \mu[n] \]

**Step 3. Zero state response**

\[ y[z][n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m] \mu[n-m] \]

\[ = \sum_{m=-\infty}^{\infty} 4^{-m} \mu[m] \left[ (0.2)^{n-m} + 4 \times (0.8)^{n-m} \right] \mu[n-m] \]

\[ = \sum_{m=0}^{\infty} 4^{-m} \left[ (0.2)^{n-m} + 4 \times (0.8)^{n-m} \right] \]

\[ = \sum_{m=0}^{n} (-0.2)^{n-m} \left[ 4 \times (-0.2)^m + 0.8^n \right] + \sum_{m=0}^{\infty} (0.8)^n \left[ 4 \times (0.8)^m \right] \]

\[ = (-0.2)^n \sum_{m=0}^{n} (-0.8)^{-m} + (0.8)^n \sum_{m=0}^{\infty} (3.2)^{-m} \]

\[ = (-0.2)^n \frac{1}{1 - (-0.8)} + (0.8)^n \frac{1}{1 - (3.2)} \]

\[ = (0.2)^n \frac{1 - (-0.8)^{n+1}}{1 - (-0.8)} + (0.8)^n \frac{1 - (3.2)^{n+1}}{1 - (3.2)} \]

\[ = (1.26 (4)^{-n} + 0.444 (-0.2)^n + 5.81 (0.8)^n) \mu[n] \]

**Step 4. Total response**

\[ y[tot][n] = y[z][n] + y[2] = \]

\[ = c_1 (-0.2)^n + c_2 (0.8)^n \left[ -1.26 (4)^{-n} + 0.444 (-0.2)^n + 5.81 (0.8)^n \right] \mu[n] \]
Using initial conditions

\[ y_{in}[-1] = C_1 \cdot (0.2)^{-1} + C_2 \cdot (0.8)^{-1} = 0 \]
\[ y_{in}[0] = C_1 + C_2 + (-1.26 + 0.444 + 5.81) = 2.5/4 \]

Solving for \( C_1 = 0.2 \) and \( C_2 = 0.8 \)

Final answer:

\[ y_{res}[n] = 0.2 \times (0.2)^n + 0.8 \times (0.8)^n \]
\[ -1.26 \cdot (4)^{-n} + 0.444 \cdot (-0.2)^n + 5.81 \cdot 0.8^n, \ n \geq 0 \]

Summary:

Step 1: Determine zero input response (Use the constants)

Step 2: Determine impulse response of the system

Step 3: Determine zero state response as a convolution of input signal and impulse response

Step 4: Determine total response and use initial conditions to determine the values of the constants.
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