Lecture 7: Waveform Coding

- Designed to reproduce input waveform with smallest possible distortion
- No attention paid to the mechanism that produces waveform (look at the voice signal)
- Waveform codes are robust and can be used with variety of source signals with predictable performance characteristics

Some basic waveform methods:

- Pulse-Encoded Modulation (PCM)
- Differential Pulse-Encoded Modulation (DPCM)
- Delta Modulation (DM)
- Adaptive Delta Modulation (ADM)

**Pulse-Encoded Modulation**

Black diagram of a PCM system with uniform quantization:

- Presampling filter 
- Presampling filter implies for BW of the signal to ensure applicability of sampling theorem

1. If quantizer is uniform $\rightarrow$ Uniform PCM
2. If quantizer is nonuniform $\rightarrow$ Nonuniform PCM

- Choice of quantizer is based on the characteristics of the input signal
- Most of the natural signals require nonuniform quantizer
Uniform PAM

- Range of input samples: $[-U_{min}, U_{max}]$
- HP range is symmetric: $[-U_0, U_0]$
- The number of quantization levels $q = 2^n$ is usually assumed as a power of 2

Quantization step: $\Delta = \frac{2U_0}{2^n} = \frac{2U_0}{q} = \frac{U_0}{2^{n-1}}$

Power of quantization noise:

$$P_{sq} = \frac{\Delta^2}{12} = \frac{(U_0/2^{n-1})^2}{12} = \frac{U_0^2}{12 \times 2^{2n-2}} = \frac{U_0^2}{3 \times 2^{2n}}$$

$n$ - number of bits per sample.

SQR

$$SQR = 10 \log \left( \frac{E_s x^2}{P_{sq}} \right) =$$

$$= 10 \log \left( \frac{E_s x^2}{P_{sq}} \right) = 10 \log \left[ \frac{E_s x^2}{U_0^2} \right] + 10 \log (2^{n+1}) + 10 \log (3)$$

$$= P_n [\text{dB}] + 6n + 4.8 \quad [\text{dB}]$$

where $P_n [\text{dB}] = \text{normalized power of the input signal}$.

- $P_n [\text{dB}]$ - depends on the shape of the signal
- Maximum value of $P_n [\text{dB}]$ is when the input signal is wavenumber clustered through the range $[-U_0, U_0]$ in worst case $P_n dB \approx -4.8 \text{ dB}$ and

$$SQR \approx 6n [\text{dB}]$$
Nonuniform PCM:

- Range of input signals $[-U_{min}, U_{max}]$
- If the range is uniform $[-U, U]$
- The number of quantization levels $R = 2^n$, $n$ - number of bits
- Quantization step is nonuniform

\[ \Delta(x) = \frac{2U}{2^n F(x)} \]

Power of quantization noise:

\[ P_{eq} = \frac{1}{12} \left( \frac{U}{2^{n+1}} \right)^2 \int_{x_{min}}^{x_{max}} \frac{pdf(x)}{F(x)} \, dx = \]

\[ = \frac{U^2}{3 \cdot 2^n} \int_{x_{min}}^{x_{max}} \frac{pdf(x)}{F(x)} \, dx \]

\[ SNR = 10 \log \left( \frac{E \cdot x^2 / P_{eq}}{P_{eq}} \right) = \]

\[ = 10 \log \left( \frac{E \cdot x^2}{P_{eq}} \right) - 10 \log \left[ \int_{x_{min}}^{x_{max}} \frac{pdf(x)}{F(x)} \, dx \right] \]

\[ = P_{eq} [\text{dB}] + 6n - 4.8 - 10 \log \left[ \int_{x_{min}}^{x_{max}} \frac{pdf(x)}{F(x)} \, dx \right] \]

\[ = P_{eq} [\text{dB}] - 1 \log \left[ \int_{x_{min}}^{x_{max}} \frac{pdf(x)}{F(x)} \, dx \right] + 6n - 4.8 \]

For a given signal and given $F(x)$, this is constant

Rule: 6dB SNR increase per every added bit still holds.
Data rate of a PCM signal

\[ R_{pcm} = f_s \times n \]

- \( f_s \): Sampling rate
- \( n \): number of bits per sample

Bandwidth of PCM signal (To be demonstrated later)

\[ B_W = f(R_{pcm}, b_{bit}, R(k)) \]

- \( R_{pcm} \): rate of the PCM signal
- \( b_{bit} \): impulse response of the pulse shaping filter
- \( R(k) \): auto-correlation function of the bit stream

In practice: \( B_W \approx R_{pcm} \) for baseband PCM

Example: Consider a PCM for a signal with bandwidth of \( W = 4 \text{kHz} \) that uses 8 bits per sample.

- Sampling rate: \( f_s = 2 \times W = 2 \times 4 \text{kHz} = 8 \text{kHz} \) (\text{samples/sec})
- Rate: \( R_{pcm} = 8 \text{kHz} \times 8 = 64 \text{kbps} \)
- \( B_W = 64 \text{kHz} \)

Time Division Multiplexing of PCM signals

- PCM modulation \( \rightarrow \) due to sampling \( \rightarrow \) discontinuities in time domain
- digital \( \rightarrow \) synchronization necessary

allows whitening of the same line in TDM mode
**Principle of Digital TDN (synchronous implementation)**

1. **TX**
   - Analog to PCM
   - $w_1, w_2, w_3, \ldots$ to $R_{bo}$

2. **RX**
   - Analog to PCM
   - $w_1, w_2, w_3, \ldots$ to $R_{bo}$

3. **Mix**
   - $w_{in}$
   - multiplexed line

4. **Denux**
   - $R_{bo}$

- $w_{ij}$ is the $j^{th}$ word of the $i^{th}$ user
- One word corresponds to a sample: $R_{bo} = N \times R_{bo}$

* PCH bit streams of multiple users are multiplexed on a single high capacity line.
* The rate on the multiplexed line is $N \times R_{bo}$ where $R_{bo}$ is the rate of the single PCH channel.

* PCH TDN systems are used in longline telephony.
* There are 2 types of commercially available systems:
  - "American" (TI - Lucordenay) (US, Canada, Japan)
  - "European" (EI - Lucordenay) (Europe, majority of the world)

* Telecommunications equipment of most big vendors supports both TI & EI standards. Usually they are just 2 pads on the "common wire
cable".

**Elements of TI - PCH**

* TI - PCH TDN multiplexes 24 channels on a single line.
* Each channel is commonly referred to as the DSO.
* One multiplex of 24 channels is called TDN Frame.
Structure of T1 TDM Frame

\[ T = \frac{1}{8 \times 10^3} = 0.125 \text{ ms} \]

Channel

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| F | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 | W11 | W12 | W13 | W14 | W15 | W16 | W17 | W18 | W19 | W20 | W21 | W22 | W23 | W24 |

\[ \text{No. of bits} = 24 \times 8 + 1 = 193 \]

\[ T_b = \frac{0.125 \text{ ms}}{193} = 0.6477 \mu \text{s} \]

- One bit used for frame synchronization.
- Word used per channel is 8 bits long and it comes voice + signaling.
- There are 2 types of frames: "robbed" and "non-robbed" frames.

**Robbed Frame**

Channel

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>7+1</td>
<td>7+1</td>
</tr>
</tbody>
</table>

- 3 Sync bit
- 7 bits for voice (\( \mu = 255 \), 8 bit encoding, 158 robbed)
- 1 bit for signaling

**Non-robbed Frame**

Channel

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

- 3 Sync bit
- 8 bits for voice (\( \mu = 255 \), 8 bit encoding)

Homework

| 6.53 |
| 6.59 |

12 Frames form a superframe

Structure of the superframe:

Frame number: 0 1 2 3 4 5 6 7 8 9 10 11 0

\[ R_{TH} = \frac{1}{T_b} = \frac{1}{0.6477 \mu \text{s}} = 1.544 \text{ Mb/sec} \]

\[ R_{DX} = \frac{8 \text{ bits}}{12 \times 0.125 \mu \text{s}} = 16 \text{ kb/sec} \]

\[ R_{CH} = \frac{10}{12} \left( \frac{8 \text{ bits}}{0.125 \mu \text{s}} \right) + \frac{2}{12} \left( \frac{7 \text{ bits}}{0.125 \mu \text{s}} \right) = 62.7 \text{ kb/sec} \]

\[ R_{SIX} = \frac{1}{0.125 \mu \text{s}} = 8 \text{ kb/sec} \]