Generation of PSK signals

PSK signal - constant average power/constant envelope

Euclidean distance between PSK signals

\[ d_{\text{min}} = \sqrt{E_g \left[ (\cos(2\pi m/M) - \cos(2\pi n/M))^2 + (\sin(2\pi m/M) - \sin(2\pi n/M))^2 \right]}^{1/2} \]

\[ = \sqrt{E_g \left[ \cos^2(2\pi m/M) + \cos^2(2\pi n/M) - 2 \cos(2\pi m/M) \cos(2\pi n/M) + \sin^2(2\pi m/M) + \sin^2(2\pi n/M) - 2 \sin(2\pi m/M) \sin(2\pi n/M) \right]}^{1/2} \]

\[ = \sqrt{E_g \left[ 2 - 2 \cos \left( \frac{2\pi (m-n)}{M} \right) \right]} = \sqrt{E_g \left[ 4 \sin^2 \left( \frac{\pi (m-n)}{M} \right) \right]}^{1/2} \]

\[ = \sqrt{E_g \left| \sin \left( \frac{\pi (m-n)}{M} \right) \right|} \]

Depends on M.
Of a special interest is the case when \(|w-n|=1\) to adjacent symbols

\[ H=2 \quad d = \sqrt{E_g} \left| 2 \sin \left( \frac{\pi}{2} \right) \right| = 2 \sqrt{E_g} \]

\[ H=4 \quad d = \sqrt{E_g} \left| 2 \sin \left| \frac{\pi}{4} \right| \right| = \sqrt{2} \cdot \sqrt{E_g} = 1.414 \sqrt{E_g} \]

\[ H=8 \quad d = \sqrt{E_g} \left| 2 \sin \left| \frac{\pi}{8} \right| \right| = 0.7071 \sqrt{E_g} \]

\[ H=16 \quad d = \sqrt{E_g} \left| 2 \sin \left| \frac{\pi}{16} \right| \right| = 0.3962 \sqrt{E_g} \]

Notes:
* If the energy is kept constant, the distance between symbols decreases as \(H\) grows.
* If the distance is to be kept the same, the energy must increase.
* For higher modulation schemes (larger \(H\)) there are more bits per symbol.
* For more bits per symbol, the symbol rate increases.
* BW of modulated signal depends on symbol rate.

Two dimensional signals - Quadrature Amplitude Modulation (QAM)

General form of a QAM signal is given by:

\[ u_m(t) = \text{A}_m \cos \left( \omega_m t + \phi_m \right) \]

Amplitude phase \(u_m(t)\) width \(A_m\) frequency \(f_m\) channel number \(n\) time

Consider

\[ u_{m1}(t) = A_m \cos \left( \frac{\pi}{2} \right) \cos \left( \omega_m t + \phi_m \right) - \sin \left( \frac{\pi}{2} \right) \sin \left( \omega_m t + \phi_m \right) = A_m \cos \left( \frac{\pi}{2} \right) \cos \left( \omega_m t + \phi_m \right) - A_m \sin \left( \frac{\pi}{2} \right) \sin \left( \omega_m t + \phi_m \right) \]

\[ = A_m \cos \phi_m \left( \cos \left( \omega_m t \right) + \sin \left( \omega_m t \right) \right) - A_m \sin \phi_m \left( \sin \left( \omega_m t \right) - \cos \left( \omega_m t \right) \right) \]

\[ = \sqrt{E_g} \cos \left( \omega_m t \right) - \sqrt{E_g} \sin \left( \omega_m t \right) \]
\[ \text{Num}\ H_1 = \text{Am}\cos(\theta) \sum_{\theta} E_k \Psi_{1k} + \text{Am}\sin(\theta) \sum_{\theta} E_k \Psi_{2k} \]

Equivalent signal representation

\[ \text{Num}\ H_1 = \sum_{E_k} \left[ \text{Am}\cos(\theta) \Psi_{1k} + \text{Am}\sin(\theta) \Psi_{2k} \right] \]

Notes

* Total number of points in constellation is \( N_t = N_1 N_2 \)

\( H_1 \) - possible coordinates along \( \Psi_1 \)

\( H_2 \) - possible coordinates along \( \Psi_2 \)

* Number of bits per symbol \( k = \log_2 (N_t) = \log_2 (N_1 N_2) \)

Some examples:

<table>
<thead>
<tr>
<th>4-QAM (QPSK)</th>
<th>( N_1 = 2 )</th>
<th>( N_2 = 2 )</th>
<th>8-QAM</th>
<th>( N_1 = 4 )</th>
<th>( N_2 = 2 )</th>
</tr>
</thead>
</table>

\[ \text{16-QAM} \quad H_1 = 4, \ H_2 = 4 \]

* QAM constellation is popular in modern design

* Constellations may be large as 2048 QAM

* Used in 802.11, 802.20, ...
Generation of QAM Signal

Am, Os[on] → Amc
Am, S[10] → Amc

Reception of Digitally Modulated Signals in AWGN Channel

\[ \text{Switch} \rightarrow \text{Rf} \rightarrow \text{IF} \]
\[ n_{\text{IF}} \]
\[ \mathcal{N} = 1, 2, 3, \ldots, M \quad \text{M - number of symbols (waveforms)} \]
\[ \mathcal{N} \in (0, 1) \quad \text{T - symbol interval} \]

Switch - one of possible M waveforms (usually \( M = 2^k \), \( k \) - number of bits per symbol)
\( n_{\text{IF}} \) - Gaussian noise \( n_{\text{IF}} \sim \mathcal{N}(0, \sigma^2) \)

\( T \) - Signaling period (\( T \)-symbol duration, \( \frac{1}{T} \) - symbol rate, \( k \cdot \frac{1}{T} \) - bit rate)

No - single sided PSD of the noise

In the noise domain the effect of the noise may be seen as perturbation of the constellation point. From \( \text{Sum} \) to \( \text{Rf} \)
The RX needs to perform two tasks:

1) Demodulation - converts the RX waveform into a N-component vector, i.e.
   \[ r(t) \rightarrow y = [y_1, y_2, \ldots, y_N] \]

   The RX projects the received signal along basis signals.

2) Detection - decision is made on which one of N possible waveforms was sent by
   the transmitter.

Demodulation - Correlation type:

\[ r(t) = s(t) + n(t), \text{ received signal} \]

\[ r_i(t) \rightarrow \mathbf{y} = [y_1, y_2, \ldots, y_N] \]

\[ r_k = \langle r(t), \psi_k(t) \rangle = \int_0^T r(t) \cdot \psi_k(t) \, dt = \]

\[ = \int_0^T (s(t) + n(t)) \cdot \psi_k(t) \, dt = \]

\[ = \int_0^T s(t) \cdot \psi_k(t) \, dt + \int_0^T n(t) \cdot \psi_k(t) \, dt = \]

\[ = s_{yk} + n_k, \quad k = 1, 2, \ldots, N \]

\( s_{yk} \) - coordinate of the signal along the vector \( \psi_k \);
\( n_k \) - noise component along the vector \( \psi_k \).
Outline of the correlation receiver

\[ Y_1 = S \sin(\omega_1 t + \phi_1) \]
\[ Y_2 = S \sin(\omega_2 t + \phi_2) \]
\[ R_n = S \sin(\omega_n t + \phi_n) \]

\[ (Y_1(t), Y_2(t)) \] - each branch produces a dot product

**Fundamental problems:** Synchronization (beyond the scope here - addressed in the textbook)

2) Demodulation - Hatched filter.

\[ r(t) = \int_0^T r(t) \cdot \psi(t) dt \] - operation performed by the correlator:

\[ r(t) \rightarrow h(t) \rightarrow y(t) \]

\[ y(t) = r(t) \cdot h(t) = \int_0^T r(t) \cdot h(t) \cdot dt \]

\[ y(t) = \int_0^T r(t) \cdot h(t) \cdot dt \] (2)

Using (2) and (2*) one sees that (2) may be implemented with a filter if

\[ h(t) = \psi(t) \quad \text{for } t \in (0, T) \]
If that is the case

\[ y_k(t) = \int_{-\infty}^{\infty} r(t - \tau) \psi_k(t - \tau) \, d\tau = \int_{-\infty}^{\infty} r(t - \tau) \cdot \psi_k(\tau) \, d\tau \]

and

\[ y_k(t) = \int_{-\infty}^{\infty} r(t - \tau) \psi_k(t - \tau) \, d\tau = \langle r(t), \psi_k(t) \rangle \rightarrow \text{same as (4)} \]

Filter with response \( \psi_k(t) = \psi_k(t - t) \) is called the matched filter.

![Diagram](image-url)