Thresholding clipped in FH modulation

\[(S/N)_{\text{out}} = 3(1+\beta)/\beta^2 (2P_h) \cdot (S/N)_{\text{in}}\]

\[(S/N)_{\text{out}} \text{ (dB)} = 10 \log [3(1+\beta)/\beta^2 (2P_h)] \text{ (dB)} + (S/N)_{\text{in}} \text{ (dB)}\]

\[= PS \text{ (dB)} + (S/N)_{\text{in}} \text{ (dB)}\]

\[\downarrow \text{ processing gain}\]

\[\begin{align*}
&\text{actual curves} \\
&\text{ideal curves}
\end{align*}\]

FH receiver curves

\[(S/N)_{\text{out}} = (S/N)_{\text{in}}\]

Increasing \(\beta\)

The actual curves depart from ideal.

At the point of the knee (black points in the graph), the assumption on high input SNR does not hold anymore. The performance of the system quickly degrades.

Approximate expression for the threshold

\[-(3A)_t = 13 + 10 \log (1+\beta) \text{ (dB)}\]
Example 1. Consider an FM system with \( p = 5 \). Calculate the input (S/N)_I. Calculate PG. Assume 2PFm = 1.

\[(S/N)_I = 12 + 20 \log (1 + 5) = 20.78 \text{ dB} \]

\[P_G = 10 \log \left( \frac{S}{S + 5} \right) = 26.5 \text{ dB}. \]

The processing gain at 26.5 dB is obtained only if the signal to noise ratio at the output is already about 20.78 dB.

Improvement of FM performance due to spectral shaping.

Spectral density of the noise at the output of FM demodulator is given by

\[S_{n(f)} = \frac{V_2}{S} \frac{k^2 f^2}{N_o} \text{ Hz} \]

\[N_o \text{ - PSD of the noise at the input of demodulator (one sided)} \]
\[f \text{ - Frequency} \]
\[S \text{ - Power of the FM signal (unmodulated carrier } \frac{S_0}{2} /2) \]
\[k \text{ - Constant associated with demodulator} \]

Consider an FM signal modulated with a single tone with magnitude \( V_m \)
For the signal, PE to DE combination is specially flat.
For noise, the DE changes the PSD and reduces its power.

An example of a PE circuit is given as:

\[
H(\omega) = \frac{r}{\frac{1}{R_1} + \frac{1}{j\omega C}} = \frac{V}{1 + \frac{j\omega C}{R_1}} = \frac{r}{1 + \frac{j\omega C}{R_1}}
\]

Therefore,

\[
\log(\sqrt[2]{|H(\omega)|^2}) = 2 \log \left( \frac{r}{\frac{1}{R_1} + \left( \frac{\omega C}{R_1} \right)^2} \right) = 2 \log \left( \frac{r}{R_1} \right) + \log \left( 1 + \left( \frac{\omega C}{R_1} \right)^2 \right)
\]

Note, PE may be implemented using a simple passive circuit.
\[ w(t) = A_c \cos\left(2\pi f_c t + \frac{2\pi}{f_w} \int_{-\infty}^{t} u_m \cos\left(2\pi f_m t \right) dt \right) = \]
\[ = A_c \cos\left(2\pi f_c t + \frac{2\pi}{f_w} \left( u_m \sin\left(2\pi f_m t \right) \right) \right) = \]
\[ = A_c \cos\left(2\pi f_c t + \frac{2\pi}{f_w} \left( \sin\left(2\pi f_m t \right) \right) \right) \]

\[ \Delta f - \text{maximum frequency deviation} \quad \Delta f \sim u_m \quad \text{proportional to magnitude} \]
\[ \Delta f/f_w = \beta \quad \text{modulation index} \]

**Note:**

1. PSD of the noise at the output given in \( m(x) \) is more harmful to higher frequency components in the spectrum of the message signal.
2. Most message signals have energy concentrated in lower portions of the spectrum.

\[ \text{PSD} \]

\[ \text{low PSD at higher end of the spectrum} \]

3. Higher frequencies in the signal spectrum carry most of the information.

Conclusion: We need a method to pull lower frequencies in the spectrum of the message signal. In FM system this is usually accomplished through pre-emphasis and de-emphasis filtering.
\[
\log (1 + \frac{f}{f_0})^2 = 2 \log \frac{f}{f_0} + \log (1 + \frac{f}{f_0})^2
\]

\[\text{3dB} \quad \text{20dB/dec increase.}\]

To regain the signal power, PE is usually followed by an amplifier.

A simple DE circuit is given by:

\[
\text{BE \, (A)} = \frac{X_{24} \mu F C}{D + \frac{X_{24} \mu F C}{2}} = \frac{1}{1 + j \omega_C R C} = \frac{1}{1 + j \left(\frac{c}{f_0}\right)}
\]
\[
\frac{1}{10} \log \left( \frac{1}{1 + (f/f_0)^2} \right) = -10 \log \left( 1 + (f/f_0)^2 \right)
\]

For \( \frac{f}{f_0} \ll 1 \), \( 10 \log (1) \approx 0 \text{ dB} \)

For \( \frac{f}{f_0} \gg 1 \), \( 20 \log \left( \frac{f}{f_0} \right) \to -20 \text{ dB/dec} \)

**Example 1.** PE/DE filters for commercial broadcasting

- **Prewphasis** (2nd order circuit)
- **Deemphasis**

\( f_0 = 2.1 \text{kHz} \) - up to this frequency, the components of the signal are unaffected (\( RC = 75 \mu \text{sec} \))

\( W = 15 \text{kHz} \) - bandwidth of the signal used for FH broadcasting

**Performance improvement due to PE/DE**

Performance improvement is determined by accessing the power of the noise at the output of FH demodulator.
For the signal, PE/DE combination is an "all pass filter."

\[ S_{\text{no}}(f) = \frac{1}{2} k^2 f^2 \frac{N_0}{s} \quad \text{PSD of the noise at the output} \]

\[ N = \int_{-W}^{W} S_{\text{no}}(f) \, df = 2 \int_{0}^{W} \frac{1}{2} k^2 f^2 \frac{N_0}{s} \, df = \frac{1}{3} k^2 W^3 N_0 = \text{Power of the noise} \]

When the DE is used, the power of the noise may be calculated as:

\[ S_{\text{DE}}(f) = S_{\text{no}}(f) \left| H_{\text{DE}}(f) \right|^2 = \frac{1}{2} k^2 f^2 \frac{N_0}{s} \cdot \frac{1}{1 + (f/f_0)^2} \]

\[ N_{\text{DE}} = \int_{-W}^{W} S_{\text{DE}}(f) \, df = 2 \int_{0}^{W} \frac{1}{2} k^2 f^2 \frac{N_0}{s} \frac{1}{1 + (f/f_0)^2} \, df \]

\[ N_{\text{DE}} = \frac{k^2 N_0 f_0^2}{s} \left[ \frac{W}{f_0} - \text{atan} \left( \frac{W}{f_0} \right) \right] = \text{noise after demodulation} \]

Therefore, the improvement is given by:

\[ \frac{Q = N_{\text{DE}}}{N_{\text{no}}} = \frac{1}{3} \frac{k^2 W^3 N_0}{s} \cdot \frac{W}{f_0} - \text{atan} \left( \frac{W}{f_0} \right) \]

\[ = \frac{W^3}{3} \left[ \frac{W}{f_0} - \text{atan} \left( \frac{W}{f_0} \right) \right] \]

Example. Estimate the improvement in FM broadcasting.

\[ Q = \frac{1}{3} \left( \frac{15 \cdot 10^3}{21 \cdot 10^3} \right)^3 \cdot \frac{15/24 - \text{atan} \left( 15/24 \right)}{15/24 - \text{atan} \left( 15/24 \right)} = 21.27 \rightarrow 13.27 \text{ dB} \]

PE/DE combination provides improvement \( \sim 13 \text{ dB} \). For this improvement to be realized, the input signal needs to be above \( (S/N)_T \).

**Homework (14)**

Exercise: 8.14, 8.15, 8.16

Assignment:None.