Communication Systems (Lecture 17)

Effects of noise in DSB-SC AM - coherent demodulation

\[ \begin{align*}
\text{Baseband} & \quad \hat{2}(t) \\
\text{Antenna} & \quad 2 \tilde{u}(t) \\
\text{FE Filter} & \quad \cos(2\pi f_t t) \\
\end{align*} \]

\[ U_{n(t)} = N_0 \delta(t) = A_0 \left[ 1 + u \cdot u_n(t) \right] \cos(2\pi f_t t) \quad \text{W- modulation index} \]

\[ n(t) = n_0(t) \cos(2\pi f_t t) - n_0(t) \sin(2\pi f_t t) \]

\[ \text{Power of input} = \left( A_0 \left[ 1 + u \cdot u_n(t) \right] \cos(2\pi f_t t) \right)^2 = \]

\[ = A_0^2 + \frac{u^2 A_0^2}{2} \quad \text{signal} = \frac{A_0^2}{2} + \frac{u^2 A_0^2}{2} \quad \text{noise} \]

Output of the baseband

\[ \text{Power of input} = 2W N_0 \quad \text{(same as in the case of the DSB-SC signal)} \]

The processing of the RX

\[ z(t) = \left[ A_0 \left[ 1 + u \cdot u_n(t) \right] \cos(2\pi f_t t) + n_0(t) \cos(2\pi f_t t) - n_0(t) \sin(2\pi f_t t) \right] \cos(2\pi f_t t) \]

\[ y(t) = \frac{A_0}{2} u \cdot u_n(t) + n_0(t) \]

\[ \text{Signal noise} \rightarrow \text{at the baseband} \]
Power of signal

\[ P_{\text{signal}} = \left( \frac{A_0}{2} \cdot M \cdot \delta f \right)^2 = \frac{A_0^2}{4} \cdot M^2 \cdot P_m \]

Power of the noise

\[ P_{\text{noise}} = \frac{N_0^2}{4} \cdot \delta f \cdot \delta f = \frac{N_0^2}{4} \cdot \delta f \]

Therefore, for DSB-SC AM one obtains

1. \( \frac{S}{N}_{\text{in}} = \frac{A_0^2/2 + M^2 P_m}{2N_0} \) \quad (\text{including power of the carrier})

2. \( \frac{S}{N}_{\text{out}} = \frac{A_0^2 M^2 P_m}{2N_0^2} \quad \frac{A_0^2 M^2 P_m}{2N_0} \)

From (1):

\[ \frac{S}{N}_{\text{in}} = \frac{1}{2} \cdot \frac{A_0^2 + A_0^2 M^2 P_m}{2N_0} = \frac{A_0^2}{2N_0} \left( 1 + M^2 P_m \right) \]

Therefore \( \frac{A_0^2}{2N_0} = \frac{2(S/N)_{\text{in}}}{1 + M^2 P_m} \)

and

\[ \frac{S}{N}_{\text{out}} = \frac{2M^2 P_m}{1 + M^2 P_m} \]  \( \frac{S}{N}_{\text{out}} \)

\[ \text{RX processing gain} \]
Example: Consider transmission of voice signal using DSB-SC AM modulation with following parameters:

- $M = 0.75$
- $P_m = 1$
- $f_m = 10 kHz$ ($2N$)
- Required $S/N_{out} = 35 dB$
- $N_0 = 4 \cdot 10^{-11}$ W/Hz (Rx noise figure is 10 dB)
- Rx implementing coherent demodulation.

\[ X_{[dB]} = 10 \log (X_{[W]}) \quad \text{Example:} \quad a_{[W]} = 50 \]
\[ a_{[dB]} = 10 \log (50) = 17 dB \]

\[ X_{[W]} = 10 \times X_{[dB]} \quad \text{Example:} \quad a_{[dB]} = 17 dB \]
\[ a_{[W]} = 10^{0.1 \times 17} = 50 \]

In this case, required $S/N_{out}$ in linear domain:

\[ (S/N)_{out} = 10^{3.75} = 10^{2.5} = 3.63 \]

1) Calculate required $(S/N)$_in
2) Express $(S/N)$_in in dB
3) Calculate the required signal power at the RX

For DSB-SC signal:

\[ (S/N)_{out} = \frac{2Wf_m}{1 + W^2f_m} (S/N)_{in} \]
Therefore

\[
\frac{(S/N)_m}{2} = \frac{1 + \frac{N}{P_{in}}}{2} (S/N)_{out} = \frac{1 + (0.75)^2}{2(0.75)^2} \cdot 3.162 = 4.293
\]

2) \( (S/N)_m \text{ [dB]} = 10 \log \frac{(S/N)_m}{(S/N)_{out}} = 10 \log (4.293) = 36.4 \text{ dB} \)

3) Power of the input signal

\[
(S/N)_m = \frac{\text{Power of signal}}{\text{Power of noise}} = 1393
\]

\[
\text{Power of signal} = 1393 \cdot \text{Power of noise}
\]

\[
\text{Power of noise} = \frac{2N \cdot F \cdot N_0}{2 \text{ bandwidth (BW)}}
\]

\[
\text{PSD of the noise when the NF of the receiver is included (given in the problem)}
\]

\[
\text{Power of noise} = 10 \text{kHz} \cdot 4 \cdot 10^{-7} \text{ W/N} = 4 \times 10^{-4} \text{ W}
\]

\[
\text{Power of signal} = 1.76 \cdot 10^{-6} \text{ W}
\]
Effects of noise in DSB-SC - envelope detection

\[ \text{FE filter} \rightarrow \begin{array}{c}
\text{\( y(t) = \frac{v(t)}{2} \)} \\
\text{\( \text{envelope detector} \)} \\
\text{\( \text{DC block} \)}
\end{array} \rightarrow \begin{array}{c}
\text{\( \text{y}(t) \)} \\
\text{\( n(t) \)}
\end{array} \]

\[ n(t) = A_0 (1 + w_{n(t)}) \cdot \cos(2\pi f_t t) \]

\[ n(t) = n(t) \cdot \cos(2\pi f_t t) - n(t) \cdot \sin(2\pi f_t t) \]

After the FE filter, the signal is given as:

\[ r(t) = A_0 (1 + w_{n(t)}) \cdot \cos(2\pi f_t t) + n(t) \cdot \cos(2\pi f_t t) - n(t) \cdot \sin(2\pi f_t t) \]

\[ = \left[ A_0 + w_{n(t)} \right] \cdot \cos(2\pi f_t t) - n(t) \cdot \sin(2\pi f_t t) \]

Try identity

\[ A \cos t + B \sin t = (A^2 + B^2)^{1/2} \cdot \cos (t - \arctan \frac{B}{A}) \]

Therefore

\[ r(t) = \left( \left( A_0 + w_{n(t)} \right) + n(t) \right) \cdot \cos \left( 2\pi f_t t \right) - n(t) \cdot \sin \left( 2\pi f_t t \right) \cdot \left( \frac{\text{n(t)}}{A_0 + w_{n(t)} + n(t)} \right) \]

We shall assume \( A_0 \gg n(t), A_0 \gg n(t) \)

Then, the envelope of the signal becomes

\[ e(t) = A_0 \left[ 1 + w_{n(t)} \right] + n(t) \]

After removing the DC component, signal becomes:
\[ y(t) = u(t) + r(t) \]

Therefore

\[ (S/N)_{\text{out}} = \frac{u^2 A_0^2 P_{\text{in}}}{2 W_0} \]

\[ (S/N)_{\text{in}} = \frac{A_0^2 (1 + u^2 P_{\text{in}})}{4W_0} \]

Finally, one obtains

\[ (S/N)_{\text{out}} = \frac{2 u^2 P_{\text{in}}}{1 + u^2 P_{\text{in}}} (S/N)_{\text{in}} \] which is the same expression as in the case of coherent downconversion.

**Conclusion:** For high input (S/N) ratios, the envelope detector performs similarly to the coherent downconverter. When the (S/N) in is small, the effects of noise become highly nonlinear & quality of the signal degrades rapidly.